UDK 336.781.5

KRIVA PRINOSA

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Rezime

Kriva prinosa je odnos između kamatne stope (ili troškova kredita) i vremena do dospeća duga za datog zajmoprimca u datoj valuti. Po definiciji, ne postoji ni jedna kriva prinosa koja opisuje troškove finansiranja za sve učesnike na tržištu. Najvažniji faktor za određivanje krive prinosa je valuta na koju su denominovane hartije od vrednosti. Unutar iste valute, različite institucije uzimaju novac na zajam po različitim stopama, zavisno od svog kreditnog rejtinga. Mada su detalji metodologije konstruisanja svojstveni za svaku investicionu banku, postoji konvencija koje se pridržavaju svi kada je reč o izboru instrumenata i opštih principa konstruisanja. U ovom materijalu opisano je nekoliko metodologija konstruisanja, od standardne "svop" krive do bazne krive.

UDC 336.781.5

YIELD CURVE CONSTRUCTION METHODOLOGY

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Summary

Yield curve is the relationship between the interest rate (or cost of borrowing) and the time to maturity of the debt for a given borrower in a given currency. By its definition, there is no single yield curve describing the cost of funds for all market participants. The most important factor in determining a yield curve is the currency in which the securities are denominated. Within the same currency, different institutions borrow money at different rates, depending on their credit rating. Even though the particulars of construction methodology are proprietary to each investment bank, there is a convention followed by all when it comes to choice of instruments and general construction principles. In this paper several construction methodologies are described from the standard "swap" curve to the basis curve.

Kada se porede različiti finansijski instrumenti često se koriste izrazi "implicirana kamatna stopa" ili "prinos". To je mera profitabilnosti investicionog instrumenta. Investitori zasnivaju svoje odluke na tome koliko će prinosa određena hartija od vrednosti doneti u poređenju sa drugim proizvodima na tržištu. Prinosi za različite ročnosti (vremenske periode) potrebni su investicionim bankama i drugim emitentima hartija od vrednosti da bi odredili cenu svojih proizvoda i izračunali sadašnju vrednost (SV) budućih tokova novca. U ovu svrhu se konstruišu krive prinosa korišćenjem likvidnih finansijskih instrumenata kojima se trguje na tržištu. Otuda, kriva prinosa predstavlja odnos između kamatne stope (ili troškova kredita) i vremena do dospeća duga za datog zajmoprimca u datoj valuti. Po definiciji ne postoji jedna kriva prinosa koja opisuje troškove sredstava za sve učesnike na tržištu. Najvažniji faktor za određivanje krive prinosa je valuta na koju su denominovane hartije od vrednosti. U okviru iste valute, institucije uzimaju novac po različitim stopama, zavisno od svog kreditnog rejtinga. Ovaj rad će se usredsrediti na primere izvedene sa finansijskih tržišta Velike Britanije, ali isti principi se koriste širom sveta. U Velikoj Britaniji banke visoke kreditne sposobnosti se zadužuju jedna kod druge po LIBOR-u (London International Borrowing Rate). Tako one konstruišu svoje krive prinosa (poznate i kao "svop krive") na osnovu instrumenata povezanih sa LIBOR-om. Drugi učesnici na tržištu, kao što su privredne firme, obično moraju da se zadužuju po višim stopama (na pr. spred iznad LIBOR-a). Krive prinosa za privredu ("osnovne krive") često se kotiraju kao "kreditni spred" ili "baza" iznad relevantne svop krive.

Hartije od vrednosti različite ročnosti (od prekonoćnih do 30-godišnjeg svopa) koriste se za izračunavanje stopa na njihova kuponska plaćanja (ako ih ima) i tačaka dospeća a matematička kriva se vuče kroz njih. Ovo omogućava da se interesna stopa izračuna u bilo kojoj tački u budućnosti. Mada su pojedinosti metodologija za konstruisanje svojstvene svakoj investicionoj banci, postoji konvencija koje se pridržavaju svi kada je reč o izboru instrumenata i opštih principa konstrukcije. U ovom radu opisano je nekoliko metodologija konstrukcije, ali one nisu niukoliko jedine.

Izbor finansijskih instrumenata za konstruisanje krive prinosa

U investicionim bankama usvojene krive prinosa imaju za cilj da pruže jedinstveni izvor stopa za određivanje cena proizvoda svih ročnosti. Instrumenti koji se koriste za konstruisanje krive jesu:

1. Novčani depoziti
2. Fjučersi na kamatne stope
3. Svopovi kamatnih stopa

Novčani depoziti su likvidni instrumenti kojima se trguje po trenutnim cenama. Tako da nema neizvesnosti, jer su stope fiksne i poznate svim učesnicima na tržištu. Ročnosti su kratke i kreću se od prekonoćnih kredita do jedne godine.

Fjučersi na kamatne stope su hartije od vrednosti kojima se trguje na berzi i koje nude veliku likvidnost i transparentnost cena. Broj ugovora raspoloživih za trgovinu uvek je isti (kada jedan ugovor istekne, drugi se uvodi - proces rolovera) a datumi isticanja su fiksirani (treća sreda u mesecu isporuke; s tim što su meseci isporuke mart, jun, septembar i decembar). Ovo čini fjučerse idealnim za izračunavanje prinosa. Za valute za koje ne postoje fjučers ugovori kojima se trguje na berzi, moraju da se koriste FRA (Forward Rate Agreements, ugovori o budućoj razmeni - ekvivalent za fjučerse kojima se trguje van berze). Da bi bili pogodan substitut, odabrani su 3-mesečni FRA koji počinju na IMM datume (International Money Market datumi kada se fjučers ugovori saldiraju), uzimajući u obzir prilagođavanje *konveksnosti* (videti kasnije). To je datum poslednjeg dana za trgovinu sa fjučers ugovorom na kamatnu stopu. Za sve ugovore izuzev GBP, to je definisano kao "dva radna dana pre treće srede u mesecu isporuke" s tim što su meseci isporuke mart, jun, septembar i decembar. Poslovni dani su radni dani u zemlje gde je sedište glavne berze za ugovor.

Svopovi na kamatne stope se koriste za dugoročne transakcije za koje ne postoje fjučers ugovori. Oni imaju prednost u odnosu na obveznice, jer je lakše proceniti njihov odnos prema tržišnim promenljivim stopama. Obično

When comparing different financial instruments the term "implied interest rate" or "yield" is frequently used. It is a measure of profitability of an investment instrument. Investors base their decisions on how much yield a particular security will bring compared to other products in the market. Yields for different tenors (time periods) are needed by investment banks and other security issuers to price their products and calculate present value (PV) of future cashflows. For this purpose, yield curves are constructed, using liquid market-traded financial instruments. Hence, the yield curve is the relationship between the interest rate (or cost of borrowing) and the time to maturity of the debt for a given borrower in a given currency. By its definition, there is no single yield curve describing the cost of funds for all market participants. The most important factor in determining a yield curve is the currency in which the securities are denominated. Within the same currency, different institutions borrow money at different rates, depending on their credit rating. This paper will concentrate on the examples derived form the UK financial markets, but the same principles are used world-wide. In the UK, banks with high creditworthiness borrow money from each other at the LIBOR (London International Borrowing Rate) rates. Thus they construct their yield curves (also known as "swap curves") based on LIBOR-related instruments. Other market participants, such as corporates, typically have to borrow at higher rates (e.g. spread over LIBOR). Corporate yield curves ("basis curves") are often quoted in terms of a "credit spread" or "basis" over the relevant swap curve.

Securities of different tenors (from overnight borrowing to a 30-year swap) are used to calculate rates at their coupon payment (if any) and maturity points and the mathematical curve is drawn through them. This enables yield to be calculated at any point in the future. Even though the particulars of construction methodology are proprietary to each investment bank, there is a convention followed by all when it comes to choice of instruments and general construction principles. In this paper several construction methodologies are described, but they are by no means exhaustive.

Choice of financial instruments for yield curve construction

In investment banks the adopted yield curves aim to provide single source of rates for pricing products of all maturities.

The instruments used in curve construction are:

1. Cash deposits
2. Interest Rate Futures
3. Interest Rate Swaps

Cash deposits are liquid instruments and prices are readily available. There is no uncertainty, as the rates are known at the outset. The maturities are short and range from overnight borrowing up to one year.

Interest Rate Futures are exchange-traded securities and offer great liquidity and price transparency. The number of contracts available for trading is always the same (as one contract expires, another is introduced – a rollover process) and the expiry dates are fixed (third Wednesday in the delivery month; where the delivery months are March, June, September, and December). This makes futures ideal for yield calculations. For currencies for which exchange-traded Futures contracts are not available, FRAs (Forward Rate Agreements – an over the counter equivalent to Futures) have to be used. To make a suitable substitution, 3-month FRAs that start on the IMM dates (International Money Market date when the Futures contracts are settled) are chosen, taking into account convexity adjustment (see later). It is the date of the last trading day for an interest rate future contract. For all the contracts except GBP, this is defined as "two business days prior to the third Wednesday in the delivery month" where the delivery months are March, June, September, and December. The business days referred to are business days in the country where the principal exchange for the contract is domiciled.

Interest Rate Swaps are used for longer-dated periods for which futures contracts are not available. They take precedence over bonds, as their relationship to floating market rates is much easier to estimate. "Vanilla swaps" are typically used, where fixed rate is exchanged for floating rate (LIBOR).

se koriste “Vanila svopovi” kada se fiksna stopa menja za promenljivu stopu (LIBOR).

Fjučersi imaju prednost u odnosu na sve druge instrumente. Tačka u kojoj fjučersi zamenjuju depozite i broj fjučersa koji se koriste kod svake krive zavisi od valute i instrumenata čija se cena određuje.

Na tržištima u Velikoj Britaniji podaci za konstruisanje krive dobijaju se na sledeći način:

Depoziti (stope za novac) su Libor sa tržišta u 11:00 po londonskom vremenu.

Fjučersi su cena saldiranja koju objavljuje LIFFE za evropske valute. Cene CAD i USD su sa tržišta u vreme uzimanja podatka. AUD i JPY stope su cene saldiranja na berzi.

Svopovi su zvanične kamatne stope na tržištu na kraju dana.

Tipična kriva prinosa bi se sastojala iz sledećih tačaka mreže:

Standardne tačke mreže krive prinosa		
Vrsta instrumenta	**Ročnost (datum saldiranja)**	**Stopa % (primer)**
Novac	Novac preko noći (O/N)	5.61
Novac	Novac sutra/naredni datum (T/N)	5.59
Novac	1 mesec (1M)	5.62
Novac	3 meseca (3M)	5.73
Fjučersi	Jun 2010	5.76
Fjučersi	Septembar 2010	5.79
Fjučersi	Decembar 2010	5.84
Fjučersi	Mart 2011	5.91
Fjučersi	Jun 2011	6.07
Svopovi	2 godine (2Y)	6.01
Svopovi	3 godine (3Y)	6.13
Svopovi	4 godine (4Y)	6.18
Svopovi	5 godina (5Y)	6.24
Svopovi	7 godina (7Y)	6.37
Svopovi	10 godina (10Y)	6.44
Svopovi	15 godina (15Y)	6.57
Svopovi	20 godina (20Y)	6.55
Svopovi	30 godina (30Y)	6.56

Svaka kriva uključuje *prilagođavanje konveksnosti* za fjučerse (uzima se periodično iz pouzdanog izvora za objavljivanje tržišnih podataka kao što je Blumberg), objašnjeno sa više detalja u daljem tekstu).

Standardne krive tipično modeliraju 3-mesečne kamatne stope na kredite (3-mesečni LIBOR u Velikoj Britaniji) za korišćenje u utvrđivanju cene instrumenata za sve ročnosti. Ovaj izbor je zasnovan na činjenici da bi se banka, ako joj zatreba finansiranje da bi pokrila svoju poziciju, zadužila po tim stopama. Međutim, neki praktičari na tržištu radije utvrđuju cene kratkoročnih instrumenata iz krive koja modelira 1-mesečne kamatne stope (jer bi tipično pozajmile sredstva na tržištu za kraći vremenski period). Da bi se odgovorilo ovim zahtevima, tipično se konstruišu dve krive za svaku valutu:

1. 3-mesečna kriva
2. 1-mesečna kriva

Metodologija konstruisanja 3-mesečne krive teži da koristi depozite, fjučerse i svopove.

Pri tome 1-mesečna kriva koristi samo depozite i fjučerse, jer nema potrebe za uključenjem tačaka duže ročnosti, ispuštajući na taj način svopove. Pored navedenih krivih, bazna kriva se često konstruiše i koristi za određivanje cene intervalutarnih instrumenata. Ova kriva modelira tržišne cene takvih instrumenata bliže nego kriva LIBOR-a jer koristi prave troškove finansiranja. Metodologija za sve navedene krive opisuje se u narednim paragrafima.

Metodologija konstruisanja krive prinosa

Konstruisanje 3-mesečne krive prinosa

Kao što je ranije navedeno, 3-mesečna kriva se gradi korišćenjem:

1. Depozita
2. Fjučersa
3. Svopova.

Depoziti

Stope za depozite (novac) koriste se do prvog fjučersa (ako se fjučers uključuje u krivu) ili do prvog svopa. Diskontni faktori se generišu za datum dospeća svake tačke u mreži korišćenjem standardnih formula.

Iz jednačine za forvord-forvord stopu:

$$(1 + r_{SD} \times t_{SD})(1 + r_{SD,\,ED} \times t_{SD,\,ED}) = (1 + r_{ED} \times t_{ED})$$

Futures take precedence over all other instruments. The point at which futures take over from deposits and the number of futures used in each curve depends on the currency and instruments being priced.

In the UK financial markets the curve construction data is captured as follows:

Deposits (cash rates) are Libor rates taken from Market at 11:00 London time.

Futures are the settlement prices as published by LIFFE for European currencies. The CAD and USD prices are those prevalent in the market at the time of capture. AUD and JPY rates are exchange settlement prices.

Swaps are interest rate swaps with the rates taken from the market at official End of Day (EOD) times.

Typical yield curve would consist of following grid points:

Standard Yield Curve Grid Points		
Instrument Type	**Tenor (Settlement Date)**	**Rate %(example)**
Cash	Overnight (O/N)	5.61
Cash	Tomorrow/next date (T/N)	5.59
Cash	1 month (1M)	5.62
Cash	3 months (3M)	5.73
Futures	Jun 2010	5.76
Futures	Sep 2010	5.79
Futures	Dec 2010	5.84
Futures	Mar 2011	5.91
Futures	Jun 2011	6.07
Swaps	2 year (2Y)	6.01
Swaps	3 year (3Y)	6.13
Swaps	4 year (4Y)	6.18
Swaps	5 year (5Y)	6.24
Swaps	7 year (7Y)	6.37
Swaps	10 year (10Y)	6.44
Swaps	15 year (15Y)	6.57
Swaps	20 year (20Y)	6.55
Swaps	30 year (30Y)	6.56

Each curve includes Futures *Convexity Adjustments* (taken periodically from a reliable market data dissemination source such as Bloomberg), described in more detail later.

Standard curves typically model 3-month borrowing rates (3-month LIBOR in the UK), to be used for pricing instruments of all maturities. This choice is based on the fact that should the bank require funding to cover its position it would borrow at those rates. However, some market practitioners prefer pricing short-tenor instruments off a curve that models 1-month interest rates (as they would typically only borrow funds in the market for a short period of time). To meet these requirements two curves are typically constructed for each currency:

1. 3-Month Curve
2. 1-Month Curve

The 3-Month Curve construction methodology tends to use deposits, futures and swaps.

The 1-Month curve uses only Deposits and Futures, as there is no need to include points at longer tenors, thus omitting swaps. In addition to the above curves, a Basis Curve is often constructed and used to price cross-currency instruments. This curve models market prices of such instruments closer than the LIBOR curve as it captures the true cost of funding. The methodology for all the above curves is described in subsequent paragraphs.

Yield Curve Construction Methodology

The 3-month Yield Curve Construction

As it was stated earlier, the 3-month curve is built using:

1. Deposits
2. Futures and
3. Swaps

Deposits

Deposit (cash) rates are used up until the first future (if futures are included in the curve) or the first swap.

Discount factors are generated for the maturity date of each grid point using the standard formulae.

From the equation for forward-forward rate:

$$(1 + r_{SD} \times t_{SD})(1 + r_{SD,\,ED} \times t_{SD,\,ED}) = (1 + r_{ED} \times t_{ED})$$

and the relationship between the rate and the DF:

$$d = \frac{1}{1 + r \times t}$$

i odnosa između stope i DF:

$$d = \frac{1}{1 + r \times t}$$

sledi:

$$d_{ED} = d_{SD} \times d_{SD,ED} = d_{SD} \times \frac{1}{\left[1 + r \times \frac{t_{ED} - t_{SD}}{year}\right]}$$

Gde su:

r

d_{SD} diskontni faktor za početni datum perioda
d_{ED} diskontni faktor za poslednji datum perioda
$d_{SD,\ ED}$ periodični diskontni faktor od t_{SD} do t_{ED}
r_{SD} spot stopa za početni datum perioda
r_{ED} spot stopa za poslednji datum perioda
$r_{SD,\ ED}$ kamatna stopa za period od t_{SD} do t_{ED}
i
t_{SD} početni datum za period
t_{ED} poslednji datum za period

Fjučersi

Implicirana stopa za fjučerse odnosi se na kotiranu cenu kao $f_{t1,t2}$=(100-*p*)/100+*adj* gde je $f_{t1,t2}$ forvord stopa za period koji počinje na datum isticanja fjučersa (izraženo kao procenat) i *adj* je prilagođavanje konveksnosti (opisano dalje u tekstu). Pošto je stopa fjučersa data formulom:

$$f_{t1,t2} = \left[\frac{d_{t1}}{d_{t2}} - 1\right] \times \frac{year}{t_2 - t_1}$$

gde je
t_1 početni datum perioda
t_2 poslednji datum perioda

DF za kraj perioda forvorda može se izračunati rearanžiranjem gornje jednačine:

$$d_{t2} = d_{t1} \times \frac{1}{1 + f_{t1,t2} \times \frac{t_2 - t_1}{year}}$$

DF na kraju svakog perioda forvorda mogu se naći iterativno iz cena fjučersa i DF na početku tog perioda. Da bi se započeo proces, mora biti poznat diskontni faktor na početnom datumu prvog fjučersa ("krnji diskontni faktor").

Izvođenje krnjeg diskontnog faktora

Metodologija koja je ovde opisana oslanja se na to da imamo dovoljan broj raspoloživih stopa na depozite. Da bi to funkcionisalo, neophodno je da datum saldiranja prvog fjučers ugovora (obično poznat kao "krnji datum") dolazi pre datuma dospeća poslednje stope depozita (otuda je neophodno uključiti u krivu najmanje dve novčane stope).

Posedovanje fjučers ugovora koji se saldira u vreme T_1 (tj. obuhvata period od T_1 do T_2) koje leži između dve novčane kamatne stope (recimo 3M i 6M) daje dve opcije za interpolaciju:

1. interpolisati stopu u T_1 od 3M i 6M novčanih stopa i onda izvoditi DF odatle.
2. interpolisati DF za T_1 od DF za 3M i 6M

Odluka o tome koji će se metod koristiti jednostavno je stvar izbora.

Prilagođavanje konveksnosti fjučersa

Svop ili FRA osetljivi su na dve stope:

- Terminsku stopu koja određuje varijabilnu stranu plaćanja (koja zavisi od tržišnih kamatnih stopa).
- Trenutne kamatne stope koje određuje DF za izračunavanje sadašnje vrednosti toka novca.

Nasuprot tome, cena fjučersa je osetljiva samo na terminsku stopu jer se inicijalna margina saldira unapred a dnevne fluktuacije u vrednosti fjučersa (varijabilna margina) na kraju svakog radnog dana.

Prema tome, odnos između fjučersa i terminske stope je linearan, dok je konveksan za svopove i FRA. Mada je ovaj efekat konveksnosti mali na početnom delu krive prinosa, on postaje značajniji sa rastom ročnosti.

Može se videti da je povoljno prodati fjučerse u poređenju sa kupovinom FRA. Kratka pozicija u fjučersima postaje profitabilna kada rastu kamatne stope. Varijabilne margine na ovu profitabilnu poziciju u fjučersima se mogu reinvestirati po višoj kamatnoj stopi na tržištu koja generiše još više profita.

Pošto ovo uvažavaju na tržištu, implicirana stopa na fjučerse viša je od ekvivalentne IMM FRA. Ova razlika se često zove konveksnost. Kao tržišna konvencija, prilagođavanje konveksnosti definiše se kao razlika između stope fjučersa i stope FRA.

Praktično, prilagođavanje konveksnosti se mora dodati budućoj ceni fjučersa ili oduzeti od implicirane buduće stope da bi se izvela follows:

$$d_{ED} = d_{SD} \times d_{SD,ED} = d_{SD} \times \frac{1}{\left[1 + r \times \frac{t_{ED} - t_{SD}}{year}\right]}$$

where:
r
d_{SD} is the discount factor for the start date of the period
d_{ED} is the discount factor for the end date of the period
$d_{SD,\ ED}$ is the periodic discount factor from t_{SD} to t_{ED}
r_{SD} is the spot rate for the start date of period
r_{ED} is the spot rate for the end date of period
$r_{SD,\ ED}$ is the interest rate for the period from t_{SD} to t_{ED}
and
t_{SD} is the start date for period
t_{ED} is the end date for period

Futures

Futures implied rate is related to the quoted price as $f_{t1,t2} = (100 - p)/100 + adj$ where $f_{t1,t2}$ is the forward rate for the period starting at futures expiry date (expressed as a percentage) and *adj* is convexity adjustment (described below). As the futures rate is given by the formula:

$$f_{t1,t2} = \left[\frac{d_{t1}}{d_{t2}} - 1\right] \times \frac{year}{t_2 - t_1}$$

where:
t_1 is the start date for period
t_2 is the end date for period

the DF for the end of the forward period can be calculated by rearranging the above equation:

$$d_{t2} = d_{t1} \times \frac{1}{1 + f_{t1,t2} \times \frac{t_2 - t_1}{year}}$$

The DFs at the end of each forward period can be found iteratively from futures price and the DF at the beginning of that period. In order to start the process, the discount factor at the start date of the first future (*"Stub discount factor"*) has to be known.

Derivation of the Stub Discount Factor

The methodology described here relies on having sufficient number of Deposit rates available. For it to work, it is necessary that the settlement date of the first futures contract (typically referred to as "stub") lies before the maturity date of the last deposit rate (hence it is necessary to include at least two cash rates into the curve).

Having a futures contract that settles at time T_1 (spanning period from T_1 to T_2) that lies between two cash rates (say 3M and 6M) gives two option for interpolation:

1. Interpolating rate at T_1 from 3M and 6M cash rates, and then deriving DF from it
2. Interpolating DF for T_1 from DFs for 3M and 6M

The decision on which method to use is simply a matter of choice.

Futures Convexity Adjustment

A Swap or a FRA is sensitive to two rates:

- The forward rate that determines the floating side payment.
- The spot rate that determines the DF used to calculate the present value of the cash flow.

In contrast, a future price is only sensitive to the forward rate as the initial margin is settled up-front and the variation margins are paid daily.

Thus, the relationship between future payoff and forward rate is linear, whereas it is convex for swaps and FRAs. Although this convexity effect is small at the short end of the yield curve, it becomes more significant as the maturity increases.

It can be seen that it is advantageous to be short futures compare to the purchase of a FRA. A short futures position will make money when interest rates rise. The margin received on this profitable futures position can then be reinvested at the higher interest rate prevailing in the market, generating even more profit.

As this is recognised by market practitioners, the implied Futures rate is higher than the equivalent IMM FRA. This difference is often called convexity. As a market convention, Convexity Adjustment is defined as the difference between the Futures rate and the FRA rate.

Practically, the convexity adjustment must be added to the Futures price or subtracted from the implied future rate in order to derive ekvivalentna stopa FRA.

Otuda, krive koje koriste buduće cene za konstrukciju inkorporišu ovo prilagođavanje.

Svopovi

Sa udaljavanjem krive od spot datuma, fjučers ugovori postaju manje likvidni i zamenjuju se svopovima.

Da bi se izvela jednačina za DF poslednjeg svop kupona, razmotrićemo primer dvogodišnjeg svopa gde se sadašnja vrednost (PV) budućih novčanih tokova može izračunati kao razlika između fiksnih i varijabilnih novčanih tokova:

$$PV(svop) = PV(fiksni) - PV(varijabilni) \quad (1)$$

$$PV(fiksni) = PxC_{2Y}x\alpha_{0,1}xd_1 + PxC_{2Y} x \alpha_{1,2}xd_2 \quad (2)$$

$$PV(varijabilni) = PxL_{0,1}x\alpha_{0,1}xd_1+PxL_{1,2} x \alpha_{1,2}xd_2 \quad (3)$$

gde je:

P nominalna glavnica (koja se ne razmenjuje, koristi kao osnovica za izračunavanje novčanih tokova)

d_k diskontni factor za kraj perioda $_k$, implicirajući $d_0 = 1$

$\alpha_{k-1,k}$ deo godine u kuponskom periodu ili *faktor prirasta*

$\alpha_{k-1,k} = (t_n - t_{n-1})$ / *godina*

$L_{k-1,k}$ LIBOR za taj period

C_{nY} fiksna kuponska stopa za vreme trajanja svopa.

Pošto se kamatna stopa *r* implicirana putem DF za susedne periode može se izračunati kao:

$$r_{k-1,k} = \left[\frac{d_{k-1}}{d_k} - 1 \right] \times \frac{1}{\alpha_{k-1,k}}$$

koja je ista kao LIBOR za taj period, tako da zamenjujući $L_{k-1,k}$ za $r_{k-1,k}$ u jednačini (3), umanjuje PV[varijabilni] na:

$$PV[varijabilni] = Pxd_0 - P \times d_2$$

Otuda:

$$PV\ (svop) = [PxC_{2Y}x\alpha_{0,1}xd_1+PxC_{2Y}x\alpha_{1,2}xd_2] - [Pxd_0 - Pxd_2] \quad (4)$$

Pošto je PV = 0 za svop valorizovan po paritetu (tj. vrednost fiksne i varijabilne strane je ista na početku ugovora) rearanžiranje jednačine (4) daje:

$$d_{2Y} = \frac{1 - C_{2Y}\left[\alpha_{0,1} \times d_{1Y}\right]}{1 + C_{2Y} \times \alpha_{1,2}}$$

Otuda, izračunavanje d_{2Y} zahteva poznavanje $d_{1Y.}$

Ova procedura se generalizuje dajući diskontni faktor za tačku n-godine:

$$d_{nY} = \frac{1 - C_{nY} \sum_{(i=1)}^{(n-1)Y} \alpha_{i-1,i} \times d_i}{1 + C_{nY} \times \alpha_{n-1,n}}$$

Indeks *i* u gornjoj formuli teče od prvog kupona svopa do pretposlednjeg kupona. Tako u slučaju 1-godišnjeg svopa, zbirni period je nula. Međutim da bi se primenio gornji izraz svih svop stopa i DF povezanih sa svim datumima plaćanja izuzev pretposlednjeg moraju da budu poznati ili inače interpolirani. Ovaj proces se izvodi po redu dospeća (poznato kao povezivanje ili "bootstrapping"), otuda obuhvata prvu tačku u nizu svopa.

Bootstrapping niza svopova

Ako je samo poslednji diskontni faktor (DF_n) u nizu svopova nepoznat, on se može rešiti neposredno iz poslednje jednačine. Međutim, ako ima dva ili više nepoznatih DF onda sve nepoznate moraju da se reše istovremeno (korišćenjem iterativne aproksimacije).

Metod izračunavanja DF za nove svopove u krivoj zavisi od frekvencije dve komponente svopa kao i od prisustva sintetičkih tačaka svopa (linearno interpoliranih iz tržišnih vrednosti). Sintetičke tačke se obično uvode ako fiksna komponenta ima višu frekventnost od varijabilne komponente, tako da su DF u tim međutačkama odmah raspoloživi.

U nastavku je pregled svih mogućih slučajeva:

1. *frekvencija fiksne i varijabilne komponente je podjednaka* - svaki novi svop uvodi samo jednu nepoznatu (DF_n) koja je neposredno rešiva iz poslednje jednačine.
2. *različite frekvencije, kriva koristi sintetičke tačke* - komponenta fiksnog plaćanja će sada biti

an equivalent FRA rate.

Consequently, the curves using Futures prices for construction incorporate this adjustment.

Swaps

As the curve moves away from the spot date, futures contract become less liquid and are substituted by swaps.

To derive the equation for the DF of the last swap coupon payment, we will consider an example of a two-year annual swap where the present value (PV) of the future cashflows can be calculated as the difference between the fixed and the floating cashflows:

$$PV(Swap) = PV(Fixed) - PV(Floating) \quad (1)$$

$$PV(Fixed) = P\times C_{2Y}\times\alpha_{0,1}\times d_1 + P\times C_{2Y}\times\alpha_{1,2}\times d_2 \quad (2)$$

$$PV(Floating) = P\times L_{0,1}\times\alpha_{0,1}\times d_1 + P\times L_{1,2}\times\alpha_{1,2}\times d_2 \quad (3)$$

where

P is notional principal

d_k is the discount factor for the end of period k, implying $d_0 = 1$

$\alpha_{k-1,k}$ is fraction of days in the coupon period or *accrual factor*

$\alpha_{k-1,k} = (t_n - t_{n-1})/year$

$L_{k-1,k}$ is LIBOR for that period

C_{nY} is the fixed coupon rate for the duration of the swap

As the interest rate r implied by the DFs for adjacent periods can be calculated as:

$$r_{k-1,k} = \left[\frac{d_{k-1}}{d_k} - 1\right] \times \frac{1}{\alpha_{k-1,k}}$$

which is the same as LIBOR rate for that period, so substituting $L_{k-1,k}$ for $r_{k-1,k}$ in Equation (3), reduces PV[Floating] to:

$$PV[Floating] = P\times d_0 - P\times d_2$$

Hence:

$$PV(Swap) = [P\times C_{2Y}\times\alpha_{0,1}\times d_1 + P\times C_{2Y}\times\alpha_{1,2}\times d_2] - [P\times d_0 - P\times d_2] \quad (4)$$

As PV = 0 for a swap valued at par, rearranging Equation (4) gives:

$$d_{2Y} = \frac{1 - C_{2Y}\left[\alpha_{0,1}\times d_{1Y}\right]}{1 + C_{2Y}\times\alpha_{1,2}}$$

Therefore, calculation of d_{2Y} requires knowledge of d_{1Y}.

This procedure generalizes to give the discount factor for an n-year point:

$$d_{nY} = \frac{1 - C_{nY}\sum_{(i=1)}^{(n-1)Y}\alpha_{i-1,i}\times d_i}{1 + C_{nY}\times\alpha_{n-1,n}}$$

The index i in the above equation runs from the first coupon on the swap to the last but one coupon. So in the case of a 1-year swap, the summation term is zero. However, to apply the above expression all the swap rates and the DFs associated with all but last payment date have to be known or otherwise interpolated. This process is performed in order of maturity (known as "bootstrapping"), hence it includes the first point in the swap strip.

Bootstrapping the Swap Strip

If only the last discount factor (DF_n) in the swap strip is unknown, it can be solved directly from the last equation. However, if there are two or more DFs then all unknowns must be solved for simultaneously (using iterative approximation).

The method of calculation of DFs for new swaps in the curve depends on the frequency of the two swap legs as well as the presence of the synthetic swap points (linearly interpolated from the market values). Synthetic points are typically introduced if the fixed leg has a higher frequency that the floating leg, so that the DFs at those intermediate points are readily available.

The following summarizes all possible cases:

1. *Fixed and floating leg frequency is the same* - each new swap introduces only one unknown (DF_n) which is directly solvable from the last equation.
2. *Different frequency, curve uses synthetic points* – The fixed leg payment dates will now be 'covered' by the synthetic swaps (used to calculate intermediate DFs), hence the case is directly solvable as above.
3. *Different frequency, no synthetic points* – the

"pokrivena" sintetičkim svopovima (koja se koristi za izračunavanje pomoćnih DF), otuda je slučaj rešiv neposredno kao gore.

3. *različite frekvencije, nema sintetičkih tačaka* - prva tačka svopa na tržištu daleko je od poslednjeg fjučersa i potrebne su pomoćne tačke. Ovo generiše više od jednog nepoznatog DF. Svi nepoznati DF moraju da se rešavaju istovremeno (korišćenjem iterativne aproksimacije).

U ovom slučaju prva tačka u nizu svopova (izvan niza fjučersa) izračunava se linearnom interpolacijom između poslednjeg fjučersa i prvog svopa na tržištu, ponavljanjem procesa za sve tražene tačke.

Metodi interpolacije

Linearna interpolacije zero stope

Zero stopa je kamatna stopa između dve tačke u vremenu pretpostavljajući da se sva kamata plaća kod dospeća. Pošto se zero stopa tretira kao kontinuirano-akumulirana stopa, ona se odnosi prema DF i spot stopi kao

$$\exp(-z,\alpha_{t0,t}) = DF = \frac{1}{1 + r_t\alpha_{t0,t}}$$

Gde je

$$\alpha_{t0,t} = \frac{t - t_0}{year}$$

$$z_t = -\frac{1}{\alpha_{t0,t}}\ln(DF)$$

Nepoznata zero stopa z dobija se iz linearne interpolacije unosom prave linije između dve susedne tačke z_1 i z_2:

$$z = z_1 + \frac{z_2 - z_1}{t_2 - t_1}(t - t_1)$$

ili

$$z = \frac{t_2 - t}{t_2 - t_1} z_1 + \frac{t - t_1}{t_2 - t_1} z_2$$

zamenom u prethodnoj jednačini, DF za nepoznatu tačku može da se izračuna kao:

$$DF = DF_1\hat{}\left[\frac{t - t_0}{t_1 - t_0} \times \frac{t_2 - t}{t_2 - t_1}\right] \times DF_2\hat{}\left[\frac{t - t_0}{t_2 - t_0} \times \frac{t - t_1}{t_2 - t_1}\right]$$

gde simbol **^** označava "stepen".

Log-linearna interpolacija

Log-linearna (geometrijska) interpolacija implicira da je kontinuirano-akumulirana kamatna stopa za bilo koji period unutar datog vremenskog intervala t_1 do t_2 jednaka kontinuirano-akumuliranoj kamatnoj stopi za celokupan period. Ovo znači da se u gornjoj jednačini svi faktori prirasta poništavaju i da se nepoznata DF može izračunati kao:

$$DF = DF_1\hat{}\left[\frac{t_2 - t}{t_2 - t_1}\right] \times DF_2\hat{}\left[\frac{t - t_1}{t_2 - t_1}\right]$$

koja se može rearanžirati u:

$$DF = DF_1 \times \left[\frac{DF_2}{DF_1}\right]\hat{}\left[\frac{t - t_1}{t_2 - t_1}\right]$$

Konstruisanje 1-mesečne krive prinosa

Instrumenti krive

Praktičari na tržištu koriste 1-mesečnu krivu da bi određivali cenu kratkoročnih instumenata (naročito onih sa dospećem od 1 meseca). Pošto ovi instrumenti ne zahtevaju dugoročne stope, kriva ne koristi svopove i konstruisanje se zasniva na:

1. depozitima: 1M i 3M
2. fjučersima/1MM FRA: cene fjučersa se izračavaju kao terminske stope

Konstruisanje krive

Sve tržišne stope i_t najpre se konvertuju u kontinuirano-složene stope f_t primenom:

$$\exp(f_1\alpha_{t0,t}) = 1+r_1\alpha_{t0,t}$$

ili

$$f_t = \frac{1}{\alpha_{t0,t}}\ln(1 + r_t\alpha_{t0,t})$$

Da bi se izračunale 1M stope iz 3M inputa, svaki tromesečni period pokriven fjučers ugovorom deli se na tri 1-mesečna perioda i stopa za srednji mesec $f_{i,2}$ postavi se kao jednaka stopi za ceo period f_i. Stope za ostala dva 1-mesečna perioda izračunaju se linearnom interpolacijom između dve susedne poznate stope. Stopa za srednju novčanu tačku (1M) ostaje nepomenjena, dok se stope za ostale srednje tačke iterativno prilagođavaju da bi zadovoljile jednačinu:

first market quoted swap point is too far out from the last future and intermediate points are needed. This generates more than one unknown DF. All unknown DFs must be solved for simultaneously (using iterative approximation).

In this case the first point in the swap strip (beyond the futures strip) is calculated by linear interpolation between the last future and the first market quoted swap, repeating the process for all required points.

Interpolation Methods

Zero-rate Linear interpolation

Zero-rate is the interest rate prevailing between two points in time assuming that all the interest is paid at maturity. As zero-rate is treated as continuously-compounded rate, it relates to DF and the spot rate as:

$$\exp(-z_t\alpha_{t0,t}) = DF = \frac{1}{1 + r_t\alpha_{t0,t}}$$

Where

$$\alpha_{t0,t} = \frac{t - t_0}{year}$$

$$z_t = -\frac{1}{\alpha_{t0,t}} \ln(DF)$$

The unknown zero-rate z is obtained from linear interpolation by fitting a straight line between the two adjacent points z_1 and z_2:

$$z = z_1 + \frac{z_2 - z_1}{t_2 - t_1}(t - t_1)$$

or

$$z = \frac{t_2 - t}{t_2 - t_1} z_1 + \frac{t - t_1}{t_2 - t_1} z_2$$

by substituting into the above equation, the DF for the unknown point can be calculated as:

$$DF = DF_1\hat{}\left[\frac{t - t_0}{t_1 - t_0} \times \frac{t_2 - t}{t_2 - t_1}\right] \times DF_2\hat{}\left[\frac{t - t_0}{t_2 - t_0} \times \frac{t - t_1}{t_2 - t_1}\right]$$

where the symbol ^ denotes "power of"

Log-linear Interpolation

Log-linear (geometric) interpolation implies that the continuously-compounded interest rate over any period within the given time interval t_1 to t_2 is the same as the continuously-compounded rate over the entire period. This means that in the equation above all the accrual factors cancel out and the unknown DF can be calculated as:

$$DF = DF_1\hat{}\left[\frac{t_2 - t}{t_2 - t_1}\right] \times DF_2\hat{}\left[\frac{t - t_1}{t_2 - t_1}\right]$$

which can be rearranged into:

$$DF = DF_1 \times \left[\frac{DF_2}{DF_1}\right]\hat{}\left[\frac{t - t_1}{t_2 - t_1}\right]$$

The 1-month Yield Curve Construction

Curve Instruments

1-Month curves are used by market practitioners to price short-term instruments (particularly those with 1-month tenors). Since these instruments do not require long-term rates, the curve is not using swaps and the construction is based on:

1. Deposits: 1M and 3M
2. Futures / IMM FRAs: Futures prices expressed as forward rates

Curve Construction

All the market rates i_t are first converted into continuously-compounded rates f_t using:

$$\exp(f_1\alpha_{t0,t}) = 1+r_1\alpha_{t0,t}$$

or

$$f_t = \frac{1}{\alpha_{t0,t}} \ln(1 + r_t\alpha_{t0,t})$$

In order to extract 1M rates from the 3M inputs, each 3-month period covered by the futures contract is split into three 1-month periods and the rate for the middle month $f_{i,2}$ is set to be equal to the rate for the entire period f_i. The rates for the remaining two 1-month periods are calculated by linear interpolation between two adjacent known rates. The rates for the middle cash point (1M) and the last futures point are kept unchanged, whilst the rates for the remaining middle points are adjusted in order to satisfy the equation:

$$(1+f_{i,1}\text{x}t_{i,1})(1+f_{i,2}\text{x}t_{i,2})(1+f_{i,3}\text{x}t_{i,3})=(1+f_i\text{x}t_i)$$

In other words, the effect of compounding the three 1-month rates has to be the same as the original 3-month market rate.

$$(1+f_{i,1}xt_{i,1})(1+f_{i,2}xt_{i,2})(1+f_{i,3}xt_{i,3})=(1+f_i xt_i)$$

Drugim rečima, efekat investiranja po tri 1-mesečne stope mora da bude isti kao originalna 3-mesečna tržišna stopa.

Prilagođavanje se izračunava kao razlika između dve strane jednačine podeljen brojem dana u periodu fjučersa (IMM). Prilagođavanje se primenjuje samo na srednju stopu a ostala dva se ponovo izračunavaju primenom linearne interpolacije. Ovaj proces se ponavlja dok se ne zadovolje kriterijumi konvergencije.

Konstruisanje bazne krive

Uvod

3-mesečna i 1-mesečna kriva opisane u prethodnoj sekciji čine pretpostavku da je stopa za finansiranje koja važi na tržištu LIBOR (ili ekvivalent za datu valutu). Ovo je stopa koja se koristi za finansiranje po promenljivoj stopi kao i izračunavanje DF. Međutim, zapaženo je da utvrđivanje cene za instrumente u raznim valutama iz krive zasnovana na LIBOR dovodi do neusaglašenosti u poređenju sa tržišnim vrednostima.

Ovo implicira da se tržišna stopa finansiranja razlikuje od LIBOR. Zato je potrebna druga vrsta krive za izračunavanje DF, koja bi korektno određivala cenu instrumenata na tržištu. Takva kriva se zove osnovna kriva i koristi se za izračunavanje PV svih novčanih tokova za dvovalutne instumente, dok su varijabilne komponente i dalje zasnovane na originalnoj LIBOR krivoj.

Razlog za neslaganje između cena zasnovanih na LIBOR i tržišnih cena može se objasniti činjenicom da investitor koji uzima poziciju u dvovalutnom proizvodu možda mora da ga finansira (ili hedžuje) u suprotnoj poziciji (na pr. davanje naspram uzimanja na zajam) na tržištu. Zbog činjenice da bi se hedžing ostvario kombinovanjem baznog svopa i svopa kamatne stope da bi sintetički ostvario neutralnu poziciju, ovo bi dodalo ekstra trošak dvostrukog ulaženja u tržište. Ovaj trošak se dodaje stopama dvovalutnog svopa kao spred iznad LIBOR.

Metodologija bazne krive

Bazna kriva se formira iz depozita i svopova. Fjučersi se ne upotrebljavaju, jer su to instrumenti kojima se trguje na berzi što ne zahteva dodatne troškove finansiranja, otuda je niz fjučersa zamenjen svopovima kraćih ročnosti da bi se zatvorila praznina nakon poslednje depozitne stope.

DF za baznu krivu izračunava se na sledeći način:

1. kratkoročni kraj DF se izračunava dodavanjem specificiranog spreda na LIBOR stope, da bi odrazio stvarne troškove finansiranja (baza).
2. dugoročni DF izračunavaju se iz niza svopova, istovremeno rešavajući bazne i LIBOR krive, da bi se izračunale dve nepoznate veličine (DF_n i L_n) za svaki novouvedeni svop.
3. srednjeročni DF se izračunavaju korišćenjem svopova (jer nema fjučersa), zasnovanim na LIBOR stopama (impliciranih fjučersima) i specificiranim spredovima

Sekcija koja sledi opisuje ovaj proces sa više detalja.

Kratkoročne stope (depoziti)

Novčani niz za baznu krivu sačinjava se korišćenjem sledeće formule (neznatna modifikacija izračunavanja LIBOR krive):

$$d_{ED} = d_{SD} \times d_{SD,ED} = d_{SD} \times \frac{1}{\left[1 + (r+b) \times \frac{t_{ED} - t_{SD}}{year}\right]}$$

gde je:

r važeća LIBOR stopa

b baza koja odražava povećane troškove finansiranja

godina je = broj dana u godini

d_{SD} diskontni faktor za početni datum perioda

d_{ED} diskonti faktor za poslednji datum perioda

$d_{SD,ED}$ periodični diskontni faktor od t_{SD} do t_{ED}

r_{SD} spot stopa za početni datum perioda

r_{ED} spot stopa za poslednji datum perioda

$r_{SD,ED}$ kamatna stopa za period od t_{SD} do t_{ED}

i

t_{SD} početni datum za period

t_{ED} poslednji datum za period

Dugoročne stope (svopovi)

Pošto se kriva LIBOR koristi za izračunavanje promenljive komponente novčanih tokova a bazna kriva se koristi za diskontovanje, svaki novi svop treba da zadovolji obe krive, tj. sledeće dve jednačine:

The adjustment is calculated as the difference between the two sides of the equation divided by the number of days in the futures (IMM) period. This adjustment is applied to the middle rate only, and the remaining two are again calculated using linear interpolation. This process is repeated until the convergence criteria are satisfied.

The Basis Curve Construction

Introduction

The 3-Month and the 1-Month curves described in the previous sections make an assumption that the funding rate prevailing in the market is LIBOR (or equivalent for a given currency). This is the rate used to represent floating rate funding as well as calculating DFs. However, it has been observed that pricing cross-currency instruments off the LIBOR-based curve produces discrepancies compared to market values.

This implies that the market-perceived funding rate differs from LIBOR. Hence a different type of curve is needed for calculation of DFs, which would price market-traded instruments correctly. Such a curve is called Basis Curve and is used to calculate PV of all the cashflows for cross-currency instruments, whilst the floating legs are still based on the original LIBOR curve.

The reason for discrepancy between LIBOR-based prices and the market prices can be explained by the fact the counterparty taking a position in cross-currency product may have to finance it (or hedge it) taking an opposite position (e.g. lending vs. borrowing) in the market. Due to the fact that the hedge would typically be made by combining a basis swap and the interest rate swap to synthetically create offsetting position, this would add extra cost of going into the market twice. This cost is added to the cross-currency swap rates as a basis spread over LIBOR.

Basis Curve Methodology

The Basis Curve is built from deposits and swaps. The futures are not used, as they are exchange traded instruments that do not require additional financing cost, hence the futures strip is populated with swaps of shorter maturities in order to close the gap after the last deposit rate.

The Basis Curve DFs are calculated as follows:

1. Short-end DFs are calculated by adding a specified spread to the LIBOR rates, to reflect the true cost of funding (basis).
2. Long-term DFs are calculated form the swap strip, simultaneously solving Basis and LIBOR curves, to calculate two unknown quantities (DF_n and L_n) for every new swap introduced.
3. Medium-term DFs are calculated using swaps (as there are no futures), based on the LIBOR rates (implied by futures) and the specified spreads

The sections below describe this process in more detail.

Short-Term rates (Deposits)

The Basis Curve cash strip is built using the following formula (slight modification of the LIBOR curve calculation):

$$d_{ED} = d_{SD} \times d_{SD,ED} = d_{SD} \times \frac{1}{\left[1 + (r + b) \times \frac{t_{ED} - t_{SD}}{year}\right]}$$

where:

r is the prevailing LIBOR rate

b is the basis reflecting increased cost of funding

year is = the number of days in the year

d_{SD} is the discount factor for the start date of the period

d_{ED} is the discount factor for the end date of the period

$d_{SD,ED}$ is the periodic discount factor from t_{SD} to t_{ED}

r_{SD} is the spot rate for the start date of the period

r_{ED} is the spot rate for the end date of the period

$r_{SD,ED}$ is the interest rate for the period from t_{SD} to t_{ED}

and

t_{SD} is the start date for the period

t_{ED} is the end date for the period

Long-Term rates (Swaps)

As LIBOR curve is used for calculating floating leg cashflows, and the Basis curve is used for discounting, each new swap introduced needs to satisfy both curves, i.e. the

$$\sum_{j=1}^{n} L_{j-1} \times \alpha_{j-1,j} \times DF_j = \sum_{i=1}^{n} s_n \times \alpha_{i-1,i} \times DF_i$$

za vanila svop i

$$\sum_{k=1}^{n} \left(L_{k-1} + b_n\right) \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

za bazni svop, gde su:
$\alpha_{k-1,k}$ broj dana u kuponskom periodu k
s_n kupon sa fiksnom stopom za vanila svop
$L_{k-1,k}$ LIBOR za taj period
b_n baza iznad LIBOR za konkretni svop

Sumarni faktori *i, j* i *k* namerno su različiti da bi odrazili različite frekvencije svopova i njihovih individualnih komponenata.

Zbog činjenice da se LIBOR ne koristi za diskontovanje, pojednostavljenje učinjeno u drugoj jednačini ne može biti primenjeno u prvoj. Rearanžiranje druge jednačine i uključenje prve u nju, daje za svaku ročnost *n:*

$$\sum_{i=1}^{n} s_n \times \alpha_{i-1,i} \times DF_i + \sum_{k=1}^{n} b_n \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

ili za $i = k$:

$$\sum_{k=1}^{n} \left(s_n + b_n\right) \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

Procedura bootstrappinga je potpuno ista kao i za ranije opisanu 3-mesečnu krivu i rešava neposredno samo ako je jedan DF_n nepoznat, ili istovremeno za više od jednog DF.

Srednjeročne stope (svopovi)

Da bi se kompenzovao nedostatak fjučersa u srednjeročnom delu krive, koriste se svopovi kraćih ročnosti. Međutim, oni nisu inkorporisani u krivu kako je napred opisano zbog činjenice da su stope LIBOR poznate u tim ročnostima (kako je implicirano novčanim depozitima/fjučersima korišćenim u krivoj LIBOR). Otuda DF_n (za svaki novi uvedeni svop s_n) izračunava se iz:

$$\sum_{k=1}^{n} \left(L_{k-1} + b_n\right) \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

koristeći poznate stope LIBOR L_k i unapred definisane bazne b_n.

Svaka bazna kriva će imati jednu ili više LIBOR krivih povezanih sa njom, za izračunavanje varijabilnih komponenata.

Oblik krive prinosa

Za oblik krive prinosa ima nekoliko teorija:

1. *Teorija čistog očekivanja* kaže da jedinu determinantu oblika krive prinosa čine očekivanja investitora u pogledu budućih kratkoročnih kamatnih stopa.
2. *Teorija preferencije likvidnosti* zasniva se na pretpostavci da investitori očekuju da im se kompenzuje za to što su njihova sredstva vezana za duže periode, zahtevajući stalno rastuće prinose za duže ročnosti.
3. *Teorija preferencijalnog staništa* slična je gornjoj, što implicira da investitori očekuju više prinose za duže ročnosti, jer su vezane za veći rizik. Međutim, rizik proizilazi iz likvidnosti dugoročnijih hartija od vrednosti (jer većina investitora planira kratke i srednje rokove), a ne samo iz ročnosti.

Zbog toga što kriva prinosa može da odražava očekivanja investitora u pogledu kamatnih stopa, inflacije, političkih i ekonomskih događaja kao i uticaja premije za rizik kod dugoročnijih investicija, tumačenje krive prinosa je komlikovano. Praktičari na tržištu ulažu velike napore pokušavajući da tačno razumeju koji faktori utiču na krive.

Normalna kriva prinosa

Normalna kriva prinosa implicira da prinosi rastu sa ročnošću (tj. nagib krive je pozitivan). Ovo odražava tržišna očekivanja ekonomskog rasta i povećanja inflacije u budućnosti. Tipičan odgovor centralne banke u takvom scenariju je povećanje kratkoročnih kamatnih stopa da bi se ohrabrila štednja umesto potrošnje. Postoji isto tako neizvesnost povezana sa procenama budućih kamatnih stopa i sredstvima oročenim na dugi rok. Investitori određuju cenu ovih rizika zahtevajući više prinose za dospeća u daljoj budućnosti.

Strma kriva prinosa

Navodeći se gore datom logikom, strma kriva prinosa implicira tržišna očekivanja da će ekonomija rasti po bržoj stopi u budućnosti od one u tekućem periodu, kao u vremenima ekspanzije.

Ravna ili grbava kriva prinosa

Kada sve krive imaju slične prinose,

following two equations:

$$\sum_{j=1}^{n} L_{j-1} \times \alpha_{j-1,j} \times DF_j = \sum_{i=1}^{n} s_n \times \alpha_{i-1,i} \times DF_i$$

for the vanilla swap, and

$$\sum_{k=1}^{n} (L_{k-1} + b_n) \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

for the basis swap, where:

$\alpha_{k-1,k}$ is fraction of days in the coupon period k

s_n is fixed coupon rate for the vanilla swap

$L_{k-1,k}$ is LIBOR for that period.

b_n is basis over LIBOR for the particular swap

The summation factors *i*, *j* and *k* are deliberately different to account for different frequencies of different swaps and their individual legs.

Due to the fact that LIBOR is not used for discounting, the simplification made in the second equation could not be made in the first. Rearranging the second equation and substituting the first into it, gives for each maturity *n*:

$$\sum_{i=1}^{n} s_n \times \alpha_{i-1,i} \times DF_i + \sum_{k=1}^{n} b_n \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

or for $i = k$:

$$\sum_{k=1}^{n} (s_n + b_n) \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

The bootstrapping procedure is exactly the same as for the 3-month curve, solving directly if only one DF_n is unknown, or simultaneously for more that one DF.

Medium-Term rates (Swaps)

To compensate for the lack of futures in the medium-term part of the curve, swaps of shorter maturities are used. However, they are not incorporated into the curve as described above due to the fact that LIBOR rates are known at these maturities (as implied by the cash/futures used in the LIBOR curve). Hence DF_n (for every new swap introduced s_n) is calculated from:

$$\sum_{k=1}^{n} (L_{k-1} + b_n) \times \alpha_{k-1,k} \times DF_k = DF_{spot} - DF_n$$

using known LIBOR rates L_k and predetermined basis b_n.

Each basis curve will have one or more LIBOR curves associated with it, used for calculating floating legs.

Shape of the Yield Curve

There are several theories behind the yield curve shape:

1. *The Pure Expectations Theory* states that the only determinant of the yield curve shape is investors' expectations of future short-term interest rates.
2. *The Liquidity Preference Theory* is based on the assumption that the investors expect to be compensated for having their funds tied up for long periods, requiring ever increasing yields for longer maturities
3. *The Preferred Habitat Theory* is similar to the above, implying that investors expect higher returns for longer maturities, as they are associated with more risk. However, the risk arises from liquidity of the longer term securities (as most investors plan short and medium term), rather than purely from maturity.

Because the yield curve can reflect investors' expectations of interest rates, inflation, political and economic events as well as the impact of risk premiums for longer-term investments, interpreting the yield curve is complicated. Market practitioners put great effort into trying to understand exactly what factors are driving yields.

Normal yield curve

Normal yield curve implies that yields rise with maturity (i.e., the slope of the yield curve is positive). This reflects market expectations of economic growth and rise in inflation in the future. The typical response of central banks in such a scenario is raising short term interest rates to encourage saving, rather than spending. There is also uncertainty associated with estimates of future interest rates and the funds put on long-term deposits. Investors price these risks by demanding higher yields for maturities further into the future.

Steep yield curve

Extending the logic from the above, the steep yield curve implies market expectations that the economy will grow at faster rate in the primećuje se ravna kriva prinosa. Ovo implicira neizvesnost u ekonomiji, pri čemu učesnici na tržištu odlažu svoje investicione odluke dok se situacija ne razbistri. Druga mogućnost je grbava kriva, koja se stvara kada su srednjeročni prinosi viši od kratkoročnih i dugoročnih prinosa. Ovo tipično implicira tržišna očekivanja brzog ekonomskog rasta u kratkom roku sa više neizvesnosti u budućnosti.

Inverzna kriva prinosa

Inverzna kriva prinosa predstavlja tržišna očekivanja pogoršanja ekonomije. Pored opadanja ekonomije, inverzne krive prinosa isto tako impliciraju ostajanje inflacije na niskom nivou.

Grafičko predstavljanje krive prinosa

Navedene metodologije konstruisanja krivih primenjuju se kao kompjuterski softver koji generiše krive za svaku valutu. Njihov autput je tipično skala diskontnih faktora (DF) za odabrane datume. Ako se traži DF za tačku koja nije na mreži, koristi se interpolacija da bi se obezbedila stopa ili DF za taj datum. Ali krive se često predstavljaju grafički da bi dale neku ideju o tržišnim očekivanjima, sa ročnošću na x - osi i prinosom na y - osi.

Zaključak

Nemoguće je predvideti razvoj budućih kamatnih stopa. Praktičari na tržištu koriste tekuće informacije kao najbolju procenu budućih prinosa. Metodologije za krive prinosa odražavaju njihovo poverenje u pouzdanost finansijskih instrumenata korišćenih u konstruisanju. Pošto je kriva prinosa odnos između kamatne stope i vremena do dospeća duga za datog zajmoprimca u datoj valuti, uvek će biti brojnih opcija na raspolaganju za investitora. Prudencijalni izbor koji odražava pravi trošak finansiranja je od ključne važnosti. Ovaj rad je uveo osnove konstruisanja krive prinosa koje mogu biti usvojene u svim scenarijima investiranja.

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future than in the current period, such as at times of expansion.

Flat or humped yield curve

When all maturities have similar yields, a flat yield curve is observed. This implies uncertainty in the economy, whereby the market participants are holding off their investment decisions until the situation clarifies. Another possibility is a humped curve, created when medium-term yields are higher than those of the short-term and long-term. This typically implies market expectation of rapid economic growth in the short term with more uncertainty in the future.

Inverted yield curve

The inverted yield curve represents market expectation of worsening economy. In addition to economic decline, inverted yield curves also imply that the inflation is likely to remain low.

Graphical Yield Curve Representation

The above curve construction methodologies are implemented as computer software which generates curves for each currency. Their output is typically a range of discount factors (DFs) for selected dates. If a DF is required for a non-grid point, interpolation is used to provide the rate or DF for that date. But the curves are often represented graphically to give some idea of market expectations, with the tenors on the x-axes and yields on the y-axes.

Conclusion

Evolution of future interest rates in impossible to predict. Market practitioners use the current information as the best estimate of future yields. Yield curve methodologies reflect their confidence in the reliability of the financial instruments used in construction. As the yield curve is the relationship between the interest rate and the time to maturity of the debt for a given borrower in a given currency, there will always be numerous options available to an investor. Prudent choice that reflects a true cost of funding is essential. This paper has introduced fundamentals of yield curve construction that can be adopted in all investment scenarios.