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MODELIRANJE DEVIZNOG KURSA EVRA PREMA DOLARU POMOĆU ARCH/ GARCH MODELA

Prevod
obezbedio
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Rezime

Glavni zadatak ARCH i GARCH modela je analiza vremenskih serija u kojima postoji uslovna heteroskedastičnost (vremenski promenljiv varijabilitet, odnosno nestabilnost uslovne varijanse, pojava koja se naziva volatilitet). Cilj ovih modela je da se izračuna neki od pokazatelja volatiliteta, da bi se pomoću tog pokazatelja donosile finansijske odluke. U ovom radu se ispituju performanse uopštenog autoregresionog modela uslovne heteroskedastičnosti (eng. generalized autoregressive conditional heteroscedasticity - GARCH) za modeliranje dnevnih promena logaritmovanog deviznog kursa evra prema dolaru. Primenjeno je nekoliko GARCH modela za modeliranje dnevne stope prinosa deviznog kursa evra, sa različitim brojem parametara. Za ocenjene GARCH modele karakteristično je da su dobijeni koeficijenti kvadriranih reziduala na docnji i koeficijenti uz parametar uslovne varijanse u jednačini uslovne varijanse uglavnom naglašeno statistički značajni. Zbir vrednosti ocene ova dva koeficijenta je blizu jedinice, što je tipično za GARCH modele koji se primenjuju na podatke prinosa finansijske aktive. To znači da će šokovi u jednačini uslovne varijanse biti dugotrajni. Velika vrednost zbira ova dva koeficijenta pokazuje da visoka stopa pozitivnog ili negativnog prinosa dovodi do velikih prognoziranih vrednosti varijanse u produženom periodu. Najbolje rezultate u modeliranju stope prinosa evra pokazao je asimetrični EGARCH(1,1) model. Koeficijent asimetrije u jednačini volatiliteta kod ovog modela je negativan, i nije statistički signifikantan. Negativna vrednost ovog koeficijenta sugeriše da pozitivni šokovi manje utiču na uslovnu varijansu u budućem periodu nego negativni šokovi. Asimetrični EGARCH(1,1) model obezbeđuje dokaze o leveridž efektu.

Ključne reči: Evro, volatilitet deviznog kursa, heteroskedastičnost, ARCH model, GARCH model, Leveridž efekat

JEL: C22, C32, C58, F31

MODELING THE EXCHANGE RATE OF THE EURO AGAINST THE DOLLAR USING THE ARCH/GARCH MODELS

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Summary

The analysis of time series with conditional heteroskedasticity (changeable time variability, conditional variance instability, the phenomenon called volatility) is the main task of ARCH and GARCH models. The aim of these models is to calculate some of the volatility indicators needed for financial decisions. This paper examines the performance of generalized autoregressive conditional heteroscedasticity (GARCH) model in modeling the daily changes of the log exchange rate of the euro against the dollar. Several GARCH models have been applied for modeling the daily log exchange rate returns of the euro, with a different number of parameters. The characteristic of estimated GARCH models is that the obtained coefficients of lagged squared residuals and the conditional variance parameters in the equation of conditional variance have a strong statistical significance. The sum of these two coefficients' estimates is close to a unit, which is typical for GARCH models that are applied on the data of financial assets returns. This means that the shocks in the conditional variance equation will be long lasting. The great value of the sum of these two coefficients shows that the high rates of positive or negative returns leads to a large forecasted value of the variance in the prolonged period. The asymmetrical EGARCH (1,1) model has showed the best results in modeling the euro exchange rate returns. The asymmetry term in the conditional variance equation of this model is negative and statistically significant. A negative value of this term suggests that the positive shock has less impact on the conditional variance than the negative shocks. The asymmetric EGARCH (1,1) model provides evidence of a leverage effect.

Keywords: Euro, exchange rate volatility, heteroscedasticity, ARCH model, GARCH model, leverage effect

JEL: C22, C32, C58, F31

Uvod

Fluktuacije deviznog kursa u savremenim uslovima globalizacije i liberalizacije svetske privrede značajno utiču na makroekonomske faktore kao što su: kamatne stope, cene, izvoz i uvoz, bruto domaći proizvod. Što je privreda otvorenija, snažniji je uticaj međunarodnog okruženja na promene deviznog kursa. To, međutim, ne znači da veća otvorenost implicira i veću volatilnost deviznog kursa. Odatle potiče zainteresovanost nosilaca ekonomske politike za sagledavanje potencijalne volatilnosti deviznog kursa. Učesnici u trgovini i investitori su motivisani za kvantitativno sagledavanje budućih oscilacija deviznog kursa, u cilju zaštite od 88 rizika deviznog kursa. Finansijski analitičari i ekonometričari su razvili niz ekonometrijskih modela pomoću kojih se analizira kretanje prinosa deviznog kursa. Među ovim modelima, istaknuto mesto pripada modelima autoregresione uslovne heteroskedastičnosti (eng. Autoregressive Conditional Heteroskedasticity - ARCH models) i modelima uopštene autoregresione uslovne heteroskedastičnosti (eng. Generalized Autoregressive Conditional Heteroskedasticity - GARCH models), odnosno ARCH i GARCH modelima. ARCH modele je razvio Engle (Engle, 1982), a njihovo dalje uopštavanje u vidu GARCH modela predložili su Bollerslev (Bollerslev, 1986) i Tejlor (Taylor, 1986). Do danas je predložen veliki broj asimetričnih GARCH modela, kao na primer, eksponencijalni EGARCH model, koji je formulisao Nelson (Nelson, 1991). GARCH modeli su zastupljeni u analizi finansijskih tržišta. Ovi modeli su pokazali dobre rezultate u predviđanju fluktuacija prinosa hartija od vrednosti i deviznih kurseva. Cilj ovog rada je da oceni adekvatnost GARCH modela u obuhvatanju volatilnosti deviznog kursa evra prema dolaru. To je značajno ne samo za neposredne učesnike u deviznoj trgovini već i za stanje platnog bilansa evro zone i SAD, ali i za ostale zemlje. Preostali deo rada je strukturiran na sledeći način. U drugom delu je dat pregled literature u kojoj se modeliraju devizni kursevi. U trećem delu je izložena metodologija istraživanja u ovom radu. Korišćeni podaci i empirijski rezultati istraživanja su dati u četvrtom, a zaključak u petom delu rada.

Pregled literature

Balaban (2004) je uporedio karakteristike simetričnih i asimetričnih GARCH modela na primeru serije prinosa deviznog kursa dolara i nemačke marke. Serija je modelirana pomoću GARCH(1,1), GJR-GARCH(1,1) i EGARCH(1,1) jednačina volatilnosti. Autor je utvrdio da je EGARCH model pokazao bolje rezultate od GARCH(1,1) modela u prognoziranju kretanja deviznog kursa izvan uzorka. Najlošije rezultate je imao GJR-GARCH model. Pilbeam i Langeland (2015) su istraživali koliko su jednodimenzioni GARCH modeli pogodni za prognozu volatilnosti deviznih kurseva. Posmatrana su dva perioda: prvi od 2002. do 2007. godine i drugi od 2008. do 2012. godine. Prvi period je karakterisala manja volatilnost, a drugi veća. Rezultati modela pokazuju da devizno tržište dobro ocenjuje buduću volatilnost. Prema nalazima studije, GARCH modeli su pokazali bolje rezultate u periodima niske volatilnosti. Alexander and Lazar (2006) su ispitivali mogućnosti GARCH(1,1) modela da obuhvati varijacije uslovnog koeficijenta asimetrije i koeficijenta spljoštenosti. Težište istraživanja je pokušaj da se dokaže da uopštene dvokomponentni GARCH(1,1) modeli daju bolje rezultate u modeliranju deviznog kursa od modela sa tri ili više komponenti, odnosno da su bolji od simetričnih i spljoštenih Studentovih t-GARCH modela. U istraživanju je ocenjeno 15 različitih modela kojima se modeliraju tri glavna devizna kursa dolara. Empirijski rezultati u ovom radu ne podržavaju uvođenje ograničenja u GARCH modele. Takođe je utvrđeno da su t-GARCH modeli pokazali dobre rezultate prema testu specifikacije momenta, ali da su bili inferiorniji u odnosu na NM-2 modele ((eng. normal mixture - NM) modeli koje karakteriše kombinovana normalna raspodela sa strukturom GARCH tipa)) pri modeliranju neuslovnog varijabiliteta. Sem toga, t-GARCH modeli su pokazali slabije rezultate prema ACF i VaR kriterijumu. Ghalayini (2014) je ocenio GARCH(1,1) model na primeru deviznog kursa dolara prema evru i došao do saznanja da je zbir ocenjenih ARCH i GARCH ($\alpha+\beta$) koeficijenta bio blizu jedinice, na osnovu čega je zaključio da su šokovi volatilnosti pokazali perzistentnost, što se često događa

Introduction

The exchange rate fluctuations in the contemporary phase of globalization and liberalization of the world economy have a significant impact on macroeconomic factors such as interest rates, prices, exports and imports, gross domestic product. The more the economies are open, the stronger the impact of the international environment on the exchange rate changes. This, however, does not mean that greater openness implies greater volatility of the exchange rate. Thus, the interest of economic policy for the consideration of the potential volatility of the exchange rate. The participants in trading and investors are motivated for a quantitative assessment of the future exchange rate fluctuations, in order to protect themselves against the exchange rate risk. Financial analysts and econometricians have developed a series of econometric models for the analysis of movements in the exchange rate returns. Among these models, a prominent place belongs to Autoregressive Conditional Heteroskedasticity - ARCH models and Generalized Autoregressive Conditional Heteroskedasticity - GARCH models. ARCH models were developed by Engle (Engle, 1982), and their further generalization as GARCH models was suggested by Bollerslev (1986) and Taylor (1986). To date, a large number of asymmetric GARCH models have been proposed, for example the exponential EGARCH model, formulated by Nelson (Nelson, 1991). GARCH models have been widely employed in financial markets analysis. Some good results in forecasting fluctuations in securities and exchange rates returns have been obtained by means of these models. The aim of this paper is to assess the adequacy of the GARCH models to capture the volatility of the exchange rate of the euro against the dollar. This is significant not only for the direct participants in the foreign exchange trade but also for the balance of payments situation in the Eurozone and the US, but also in other countries. The remainder of this paper is structured as follows. The review of reference literature related to exchange rates modeling is presented in the second part of the paper. The third part is devoted to the methods of research used in this paper. The fourth section presents an analysis of the empirical results, and the conclusion is presented in the fifth part of the paper.

Literature Review

Balaban (2004) compared the characteristics of symmetric and asymmetric GARCH model using a time series of the dollar and the German mark exchange rate returns. The time series were modeled with GARCH (1,1), GJR-GARCH (1,1) and EGARCH (1,1) volatility equations. The author found that the EGARCH model showed better results than GARCH (1,1) model in forecasting the exchange rate movements out of sample. The worst results were produced by the GJR-GARCH model. Pilbeam and Langeland (2015) investigated how suitable one-dimensional GARCH models are for forecasting volatility in exchange rates. The study focused on two periods: the first from 2002 to 2007 and the second from 2008 to 2012. The first period was characterized by lower volatility, the second by higher volatility. The model shows that the future volatility is well estimated by the foreign exchange market. According to the study, GARCH models have shown better results in the periods of low volatility. Alexander and Lazar (2006) examined the possibility of the GARCH (1,1) model to encompass the variations of conditional skewness and kurtosis. The research is an attempt to prove that generic two-component GARCH (1,1) models give better results in modeling the exchange rate than the models with three or more components, and that they are better than the symmetric and skewed Student's t-GARCH models. The study evaluated 15 different models which are used for modeling the three major dollar exchange rates. The empirical results of this study do not support the introduction of restrictions on GARCH models. It was also found that the t-GARCH models show good results according to the test specifications of the moment, but they were inferior to the NM-2 models (Normal mixture models that are characterized by normal distribution combined with the GARCH type structure) modeling non-conditional variability. In addition, t-GARCH models have showed the weaker results according to the ACF and VaR criteria. Ghalayini (2014) estimated GARCH (1,1) model in the case of the dollar against the euro exchange rate and came to the conclusion that the sum of the estimated

u visokofrekventnim finansijskim podacima. LM test autokorelacije je doveo do odbacivanja hipoteze o odsustvu preostale autokorelacije u rezidualima u modelu reda dva. Zaključeno je da model pokazuje prisustvo serijske korelacije, i da zbog toga ne predstavlja zadovoljavajući okvir za obuhvatanje korelacije u vremenskoj seriji, a time i za prognoziranje deviznog kursa. Hartwell (2014) je koristio familiju GARCH modela da ispita finansijsku volatilnost u funkciji od institucionalne volatilnosti. Ovi modeli su primenjeni na 20 zemalja u tranziciji u različitim intervalima vremenskog perioda 1993-2012. godine. Rezultati primenjenog EGARCH i TGARCH modeliranja podržavaju tezu da razvijenije i stabilnije institucije sprečavaju volatilnost finansijskog sektora, dok institucionalna volatilnost direktno utiče na volatilnost finansijskog sektora zemalja u tranziciji. Hsieh (2012) je ocenio ARCH i GARCH modele za 5 valuta, koristeći desetogodišnje dnevne podatke i različite specifikacije ovih modela. Takođe je primenio set obuhvatnih dijagnostičkih provera. Na osnovu sprovedenih testova, ovaj autor je zaključio da ARCH i GARCH modeli uglavnom mogu da otklone celokupnu heteroskedastičnost u promeni cena svih pet valuta. Eksponencijalni GARCH model se veoma dobro prilagodio kanadskom dolaru i švajcarskom franku, a solidne rezultate je pružio i za nemačku marku. Juraj Stančić (2006) je analizirao osnovne faktore koji utiču na volatilnost deviznog kursa novih članica EU u odnosu na evro - otvorenost privrede, faktor "vesti" i režim deviznog kursa. Modeliranje deviznog kursa je izvršeno pomoću TARARCH modela. Dobijeni rezultati pokazuju da otvorenost negativno utiče na volatilnost deviznog kursa. Takođe je utvrđeno da "vesti" značajno utiču na volatilnost deviznog kursa. Efekti ovih faktora značajno variraju između zemalja. Dumitresku and Roşca (2015) su modelirali volatilnost deviznih kurseva Rumunije, Češke, Mađarske i Poljske u periodu 2005-2014, identifikujući robustan ekonometrijski model. Dobijeni rezultati su potvrdili validnost GARCH(1,1) modela i neuslovljene volatilnosti. Mcmillan i Speight (2010) su analizirali međuzavisnost i prelivanje volatilnosti između tri devizna kursa evra (u odnosu na američki dolar, japanski jen

i britansku funtu sterlinga). Primenom metoda realizovane varijanse, ovi efekti su posmatrani u nekoliko vremenskih intervala tokom trgovačkog dana. Dekompozicija varijanse iz ocenjenog VAR modela pokazuje da devizni kurs evra prema dolaru dominira u odnosu na druga dva kursa, kako u pogledu prinosa tako i u prelivanju volatilnosti. Ocenjeno je da šokovi u kretanju deviznog kursa evra u odnosu na sterling i jen marginalno utiču na kurs evra prema dolaru, dok vesti koje se odnose na kurs evra prema dolaru objašnjavaju oko 30% kretanja prinosa i volatilnosti deviznog kursa evra prema sterlingu i jenu.

Metodologija istraživanja

Uopšteni GARCH model koji je predložio Bolderslev (1986, str. 309), poseduje sledeću specifikaciju:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i} \quad (1)$$

Model prikazan u jednačini (1) može se u najopštijem obliku označiti kao GARCH(p,q). U praktičnoj primeni je veoma popularan model prvog reda (p=q=1). Uopšteni model podrazumeva sledeće uslove: $p \geq 0$, $q > 0$, $\alpha_0 > 0$, $\alpha_i \geq 0$, pri čemu je $i = 1, \dots, q$, i $\beta_i \geq 0$, pri čemu je $i = 1, \dots, p$. Osnovna manjkavost GARCH modela je u tome što pretpostavljaju simetrične efekte pozitivnih i negativnih šokova na volatilnost. Međutim, u literaturi je rasprostranjeno uverenje da negativni šokovi u vremenskim serijama finansijskih veličina dovode do veće volatilnosti od pozitivnih udara istog obima. Kada su u pitanju serije prinosa, asimetrično reagovanje se pripisuje leveridž efektu. Zbog toga su u literaturi formulisani brojni asimetrični GARCH modeli. Jedan od njih, eksponencijalni GARCH model, primenićemo u ovom radu.

ARCH modele u osnovi čine dve jednačine: jednačina srednje vrednosti (nivo prinosa posmatrane pojave), kojom se oblikuje bezuslovna varijansa, i jednačina uslovne varijanse (volatilnosti), kojom se opisuje uslovna varijansa prinosa. Za analizu dnevnih podataka korisna je primena GARCH modela, koji uslovnu varijansu prinosa izvode u funkciji kvadrata uslovnog varijabiliteta u izabranom broju prethodnih perioda i kvadrata slučajnih

ARCH and GARCH ($\alpha + \beta$) coefficient was close to a unit, based on which he concluded that volatility shocks showed persistence, which often occurs in the high frequency financial data. The LM test of autocorrelation rejected the hypothesis of the absence of residual autocorrelation in the model residuals up to the order of two. It was concluded that the model shows the presence of serial correlation, and therefore does not represent a satisfactory framework to capture the correlation in the time series and thus cannot be used for forecasting the exchange rate. Hartwell (2014) used the family of GARCH models to examine the financial volatility as a function of institutional volatility. These models were applied to 20 countries in transition over different time intervals from 1993 to 2012. The results of applied EGARCH and TGARCH models support the view that the more developed and stable institutions prevent the volatility of the financial sector, while institutional volatility directly affects the volatility of the financial sector in transition countries. Hsieh (2012) estimated ARCH and GARCH models for 5 currencies, using the ten-year daily data for various specifications of these models. He also applied a comprehensive set of diagnostic tests. Based on the above tests, the author concluded that ARCH and GARCH models can generally remove the entire heteroskedasticity in price changes in all five currencies. The exponential GARCH model is very well fitted to the Canadian dollar and Swiss franc, and a solid performance was provided for the Germany mark. Juraj Stančík (2006) analyzed the basic factors affecting the volatility of exchange rates of the new EU member states in relation to the euro - the openness of the economy, the "news" factor and the exchange rate regime. Modelling of the exchange rate was performed by TARARCH models. The results show that openness has a negative impact on exchange rate volatility. It was also found that the "news" factor significantly affects the volatility of the exchange rate. The effects of these factors vary significantly among the countries. Dumitrescu and Roșca (2015) modeled the volatility of the Romanian, Czech, Hungarian and Polish foreign exchange rates in the period 2005-2014, identifying a robust econometric model. The

results confirmed the validity of the GARCH (1,1) model and unconditional volatility. McMillan and Speight (2010) analyzed the interdependence and volatility spillovers in the three exchange rates of the euro (against the US dollar, Japanese yen and British pound sterling). By applying the variance method, these effects are observed at several time intervals over the trading day. Variance decomposition of the estimated VAR models shows that the exchange rate of the euro against the dollar dominates in relation to the other two rates, both in terms of returns and the spillover of volatility. It is estimated that shocks in the movement of the exchange rate of the euro against the sterling and the yen affect the exchange rate of the euro against the dollar marginally, while the news concerning the euro exchange rate against the dollar account for about 30% of the movement in returns and volatility of the exchange rate of the euro against sterling and the yen.

Research Methodology

The generalized GARCH model proposed by Bollerslev (1986, p. 309) has the following specifications:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i} \quad (1)$$

The model shown in equation (1) can be in the most general form marked as GARCH (p, q). In practical applications the model of the first order (p = q = 1) is very popular. The generalized model assumes the following conditions: $p \geq 0$, $q > 0$, $\alpha_0 > 0$, $\alpha_i \geq 0$, wherein $i = 1, \dots, q$, and $\beta_i \geq 0$, wherein $i = 1, \dots, p$. The basic weakness of GARCH models is that they assume symmetric effects of positive and negative shocks to volatility. However, the belief that negative shocks in financial time series cause a greater volatility than positive shocks of the same magnitude is widespread in the literature. As regards the series of returns, asymmetric response is attributed to the leverage effect. Therefore, a number of asymmetric GARCH models have been formulated in reference literature. One of them, the exponential GARCH model, will be applied in this paper.

ARCH models basically consist of two equations: the mean equation (yield level

šokova u istom broju prethodnih perioda. Najčešće se primenjuje GARCH(1,1) model. U standardnom označavanju GARCH(1,1), prvi broj u zagradi predstavlja broj uključenih autoregresionih docnji, odnosno ARCH članove datog modela, dok drugi broj označava broj uključenih docnji pokretnih proseka, koji se u ovom modelu često označava kao broj GARCH članova (Engle, 2001, str. 160). GARCH(1,1) model može se predstaviti pomoću sledeće dve specifikacije (jednačine 2 i 3):

$$Y_t = X_t' \Phi + \varepsilon_t \quad (2)$$

gde X_t predstavlja egzogene varijable koje su uključene u jednačinu srednje vrednosti. ε_t je slučajni član modela (slučajna greška modela), koji ne poseduje normalnu raspodelu. (Imajući u vidu da su proračuni autora u ovom radu urađeni pomoću softvera Eviews, simboli u jednačinama 2 do 8, kao i prikaz GARCH modela su prema Uputstvu za program Eviews (EViews 8 User's Guide II, 2013, str. 207-208)). Jednačina srednje vrednosti može sadržati uslovnu varijansu ili uslovnu standardnu devijaciju (GARCH-in-Mean modeli). Jednačina uslovne varijanse u specifikaciji GARCH(1,1) može se prikazati kao:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3)$$

gde simboli imaju sledeće značenje: σ_t^2 je uslovna varijansa, odnosno predviđena varijansa u narednom periodu zasnovana na prošlim informacijama; ω je konstanta, ε_{t-1}^2 je ARCH komponenta modela i predstavlja informaciju o volatilnosti u prethodnom periodu, koja je izračunata kao docnja kvadriranih reziduala iz ocenjene jednačine srednje vrednosti; σ_{t-1}^2 je GARCH član modela i predstavlja prognozu varijanse za poslednji period. Opisani model je jednostavan za ocenjivanje, a njegova primena je pokazala dobre rezultate u prognoziranju vrednosti uslovne varijanse (Engle, 2001, str. 159).

Parametri ω , α i β jednačine uslovnog varijabiliteta moraju ispunjavati sledeće uslove: $\alpha + \beta < 1$, kao i $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$. Ovi uslovi obezbeđuju stabilnost bezuslovne varijanse.

Viši red GARCH modela, koji se označava kao GARCH(p, q), može se oceniti ako je p ili q veće od jedinice, pri čemu je p red pokretnog

proseka ARCH člana, a q red autoregresivnog GARCH člana. GARCH (p, q) varijansa može se prikazati kao:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (4)$$

Kada je q=0, GARCH model se svodi na ARCH model. U okviru GARCH (p, q) modela, uslovna varijansa σ_t^2 od ε_t zavisi od kvadrata reziduala u prethodnih p perioda, i uslovne varijanse u prethodnih p perioda.

Ako u jednačinu (2) (jednačina srednje vrednosti) uključimo uslovnu varijansu, dobija se GARCH-in-Mean (GARCH-M) model (Engle, Lilien and Robins, 1987). Nova jednačina srednje vrednosti glasi:

$$Y_t = X_t' \Phi + \lambda \sigma_t^2 + \varepsilon_t \quad (5)$$

Parametar uz σ_t^2 , označen kao λ , meri premiju rizika. Ako je vrednost ocenjenog koeficijenta λ pozitivna i statistički signifikantna, to znači da veći rizik ima za rezultat porast nivoa prinosa (u našem slučaju to bi značilo da će evro apresirati prema dolaru). Ako u jednačinu srednje vrednosti (5) uključujemo ocenjenu uslovnu standardnu devijaciju umesto varijanse, ARCH-M specifikacija tada se može prikazati u vidu jednačine (6).

$$Y_t = X_t' \Phi + \lambda \sigma_t + \varepsilon_t \quad (6)$$

Eksponencijalni GARCH metod je osmislio Nelson (1991). Sledeća specifikacija uslovne varijanse i njeno objašnjenje je prema Eviews 8 Users Guide (2013, str. 221, jednačina 25.22):

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^p \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \quad (7)$$

U jednačini (7) ω je konstanta (dugoročna srednja vrednost). Parametar α reprezentuje "GARCH" efekat. Parametar β meri perzistenost uslovne volatilnosti bez obzira na to šta se događa na tržištu. Parametar γ meri asimetriju ili efekat leveridža, tako da EGARCH model omogućava testiranje asimetrije. Kad je $\gamma=0$, model je simetričan, odnosno pozitivni i negativni šokovi podjednako deluju na volatilnost serije prinosa. U slučaju da je $\gamma < 0$, pozitivne (dobre) vesti sa tržišta generišu manju volatilnost nego negativni šokovi. Ako

of the observed phenomena), which forms the unconditional variance, and conditional variance (volatility) equation, which describes the conditional variance of returns. The GARCH models are useful in the analysis of daily data, and their conditional variance of return shows as a function of conditional square variability in a selected number of previous periods and the square of random shocks in the same number of the previous periods. The most commonly applied is GARCH (1,1) model. In the standard notation GARCH (1,1), the first number in parentheses refers to how many autoregressive lags, or ARCH members of a given model, are included in the equation, while the second number refers to how many moving average lags, which in this model is often referred to as the number of the GARCH terms (Engle, 2001, p. 160). GARCH (1,1) model can be represented by the following two specifications (Equations 2 and 3):

$$Y_t = X_t' \Phi + \varepsilon_t \quad (2)$$

where X_t represents the exogenous variables included in the mean equation. ε_t is a random error model, which does not have a normal distribution. (Bearing in mind that the authors' calculations in this paper are made by software EViews, symbols in equations 2 to 8, as well as the describes of GARCH model, are according to the EViews Guide (EViews 8 User's Guide II, 2013, pp. 207-208)). The mean equation may contain conditional variance or conditional standard deviation (GARCH-in-Mean models). The conditional variance equation in the specification GARCH (1,1) can be summarized as:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3)$$

where the symbols have the following meanings: σ_t^2 is a conditional variance, or forecast variance in the coming period, based on the past information; ω is constant term, ε_{t-1}^2 is a component of the ARCH model and presents the information about the volatility in the previous period, which was calculated as the lag of squared residuals from the mean equation; σ_{t-1}^2 is a member of the GARCH model and represents the forecast variance for the last period. The present model is easy to assess and its application has shown good results in

forecasting the value of the conditional variance (Engle, 2001, p. 159).

Parameters ω , α and β in the conditional variance equation must meet the following conditions: $\alpha + \beta < 1$, and $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$. These conditions provide stability of the unconditional variance.

A higher order of the GARCH model, which is referred to as GARCH (p, q) can be assessed if the p or q is greater than one, where p is the order of the moving average ARCH member, and q is the order of the autoregressive GARCH term. GARCH (p, q) variance can be summarized as:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (4)$$

When $q = 0$, GARCH model is reduced to the ARCH model. Within the GARCH (p, q) model, the conditional variance σ_t^2 of ε_t depends on the squared residuals in the previous p period, and conditional variance in the previous p period.

If conditional variance is included into equation (2) (mean equation), GARCH-in-Mean (GARCH-M) model is obtained. The new mean equation is:

$$Y_t = X_t' \Phi + \lambda \sigma_t^2 + \varepsilon_t \quad (5)$$

The parameter next to σ_t^2 , designated as λ , measures the risk premium. If the estimated value of the coefficient λ is positive and statistically significant, it means that the higher risk leads to an increase in the level of yields (in our case, this would mean that the euro will appreciate against the dollar). If the estimated conditional standard deviation were included in the mean equation instead of variance, ARCH-M specification could be displayed as equation (6).

$$Y_t = X_t' \Phi + \lambda \sigma_t + \varepsilon_t \quad (6)$$

Exponential GARCH methods were designed by Nelson (1991). The next specification of conditional variance and its explanation is according to Eviews 8 Users Guide (2013, p. 221, equation 25.22):

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^p \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \quad (7)$$

je $\gamma > 0$, pozitivni šokovi imaju veći uticaj nego negativni udari. Smer promene prinosa takođe utiče na volatilitnost. Zbir α i γ pokazuje uticaj pozitivnih šokova na seriju prinosa. Ovaj zbir će biti manji od vrednosti α kad koeficijent γ ima negativnu vrednost i obrnuto.

Na levoj strani jednakosti (7) je oznaka za logaritam uslovne varijanse. To znači da u jednačini postoji eksponencijalni leverage efekat, što garantuje da će prognoza uslovne varijanse biti nenegativna. Postojanje leveridž efekta testira se pomoću hipoteze da je $\gamma_i < 0$. Asimetričan uticaj postoji ako je $\gamma_i \neq 0$. Postoji nekoliko razlika između specifikacije EGARCH modela u Eviews-u i Nelsonovog originalnog modela. Nelson je pošao od pretpostavke da e_t sledi uopštenu distribuciju greške (eng. Generalized Error Distribution-GED), dok se u Eviews-u može izabrati normalna, Studentova t-raspodela ili GED raspodela. Međutim, zbog konceptualne lakoće i intuitivne interpretacije, pri korišćenju EGARCH modela uglavnom se primenjuje normalna raspodela grešaka (eng. conditionally normal errors). Drugo, Nelsonova specifikacija logaritmovane uslovne varijanse je ograničena verzija sledećeg modela:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^p \alpha_i \left| \frac{e_{t-i}}{\sigma_{t-i}} - E\left(\frac{e_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{k=1}^r \gamma_k \frac{e_{t-k}}{\sigma_{t-k}} \quad (8)$$

koja se neznatno razlikuje od gornje specifikacije pod (7). Model (8) daje identične ocene kao i model koji koristi Eviews izuzev ocene konstante, koja zavisi od usvojenog modela raspodele verovatnoće i reda p . Na primer, kao je $p=1$ u modelu $\frac{e_{t-1}}{\sigma_{t-1}}$ normalnom raspodelom, razlika bi bila $\alpha_1 \sqrt{2/\pi}$ (Eviews 8 Users Guide II, 2013, str. 221). Eksponencijalni GARCH model ima dve značajne prednosti u odnosu na GARCH(1,1) model. Prvo, EGARCH model meri log prinose, tako da će uslovna varijansa biti pozitivna čak i kad su parametri negativni. Zbog toga na parametre modela ne mora da se uvodi ograničenje nenegativnosti. Drugo, zbog asimetrije, model obuhvata takozvani leveridž efekat (ovaj efekat se po pravilu interpretira kao negativna korelacija između negativnih prinosa na docnji i volatilitnosti). Empirijske analize leveridž efekta podstakle su razvoj modela sa asimetričnom volatilnošću, koja nastaje usled pozitivnih i negativnih šokova. Leveridž efekat prvo je zapazio Black (1976),

a najbolje ga prikazuje kriva uticaja vesti (eng. news impact curve), koju su u analizu uveli Pagan i Schwert (1990). Ova kriva pokazuje na koji način buduća volatilitnost reaguje na dobre ili loše vesti - kod asimetričnih GARCH modela, kriva je asimetrična, tako što negativni šokovi snažnije utiču na volatilitnost od pozitivnih šokova istog intenziteta (detaljnije o uticaju vesti na volatilitnost videti kod Engle and Ng, 1993). Empirijski je takođe utvrđeno da distribucija greške utiče na ocenu parametra asimetrije, ali ne utiče na volatilitnost ostalih parametara (Rodríguez i Ruiz, 2012, str. 661).

Svi modeli u ovom radu ocenjeni su pomoću softverskog paketa EViews, uz primenu Marquardt algoritma optimizacije i Bolderslev-Vuldridžovog (Bollerslev-Wooldrige, 1992) metoda korekcije standardnih grešaka ocena. Parametri GARCH modela ocenjeni su metodom maksimalne verodostojnosti (Primena metoda maksimalne verodostojnosti sa Bolderslev-Vuldridžov-im korekcijama standardnih grešaka poznata je kao kvazi maksimalna verodostojnost (eng. quazi-maximum likelihood - QML; Brooks, 2008, str. 399)). Metod maksimalne verodostojnosti omogućava dobijanje ocena koje su asimptotski efikasnije od ocena do kojih se može doći primenom drugih metoda.

Podaci i empirijski rezultati

Podaci korišćeni u ovom radu su dnevni devizni kursevi evra prema dolaru u periodu od 03.01.2000. do 30.09.2016. godine (ukupno 4291 podatak). Korišćeni su podaci MMF-a o dnevnom promptnom deviznom kursu: <https://www.imf.org/external/np/fin/ert/GUI/Pages/CountryDataBase.aspx> Nominalni porast deviznog kursa znači apresijaciju evra prema dolaru. Polazni podaci su logaritmovani. Dnevna stopa prinosa deviznog kursa je izračunata kao $r_t = \log(y_t) - \log(y_{t-1}) \times 100$, gde je y_t nivo promptnog deviznog kursa u vreme t , pri čemu je $t=1, 2, \dots, T$. Prema ovom pristupu, *dnevna apresijacija ili depresijacija deviznog kursa evra prema dolaru dobijena je kao proa diferencija logaritmovanog nivoa deviznog kursa*. Dnevna stopa prinosa evra prema dolaru prikazana je u grafikonu 1.

In equation (7) w is an intercept term (long-term mean). The parameter α represents the "GARCH" effect. The parameter β measures the persistence in conditional volatility regardless of what is happening in the market. The parameter γ measures the effect of asymmetry or leverage, so the EGARCH model allows the testing of asymmetry. When $\gamma = 0$, the model is symmetrical, i.e. positive and negative shocks act on the return volatility equally. In the case of $\gamma < 0$, the positive (good) news from the market generate less volatility than negative shocks. If $\gamma > 0$, positive shocks have a greater impact than negative shocks. The direction of change also affects the yield volatility. The sum of α and γ shows the positive impact of shocks on the return series. This sum will be less than the value of the coefficient α when γ has a negative value, and vice versa.

On the left side of equation (7) is the logarithm of the conditional variance. It means that the leverage effect exist in exponential equation, which guarantees the forecast of the conditional variance to be non-negative. The existence of the leverage effect is tested by the hypotheses that $\gamma_i < 0$. If $\gamma_i \neq 0$ the asymmetric effect exist. There are several differences between the specifications of the EGARCH models in Eviews and Nelson's original model. Nelson started from the assumption that e_t traces to the generalized error distribution, while Eviews offers a choice between the normal, Student's t-distribution and GED distribution. However, due to the conceptual ease and intuitive interpretation, when using the EGARCH model the conditionally normal error distribution is mainly applied. Second, the Nelson's specification for the log of the conditional variance is a restricted version of the following model:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^p \alpha_i \left| \frac{e_{t-i}}{\sigma_{t-i}} - E\left(\frac{e_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{k=1}^r \gamma_k \frac{e_{t-k}}{\sigma_{t-k}} \quad (8)$$

This model is slightly different from the specifications above under (7). Model (8) gives identical estimates as well as the model that uses Eviews except for the intercept term, which depends on the adopted model, and the probability distribution and the order p . For example, as the $p = 1$ in the model with the normal distribution, the difference would

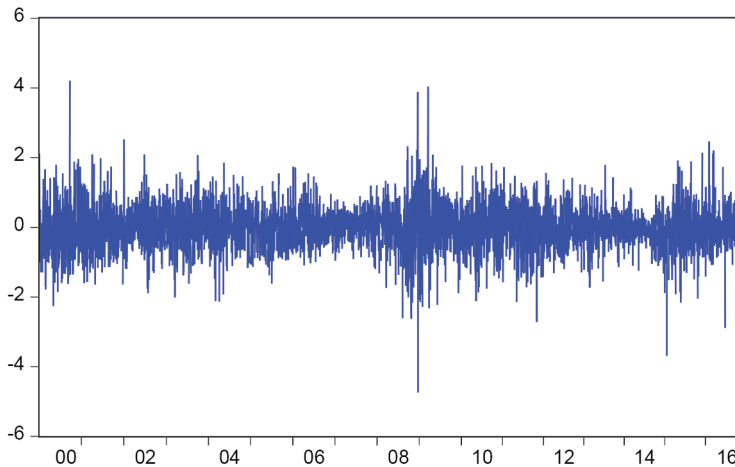
be $\alpha_1 \sqrt{2/\pi}$ (8 Eviews II Users Guide, 2013, p. 221). The Exponential GARCH model has two important advantages over GARCH(1,1) model. First, the EGARCH model show log returns, so that the conditional variance will be positive even if the parameters are negative. Therefore, non-negativity constraint does not need to be introduced on the model parameters. Second, due to the asymmetry, the model includes the so-called leverage effect (this effect is typically interpreted as a negative correlation between negative lag return and volatility). Empirical analysis of the leverage effect has prompted the development of a model with asymmetric volatility, following the positive and negative innovations. The leverage effect was first observed by Black (1976), and it is described the best by news impact curve, introduced by Pagan and Schwert (1990). This curve describes the future volatility as a reaction to good or bad news - in the case of asymmetric GARCH model the curve is asymmetric, so that negative shocks lead to higher volatility than the positive shocks of the same intensity (for more details on the impact of news on volatility see Engle and Ng, 1993). Empirically, it is also found that the error distribution affects the assessment of the asymmetry parameter, but does not affect the volatility of other parameters (Rodríguez and Ruiz, 2012, p. 661).

All models in this paper were estimated using EViews, by the Marquardt algorithm optimization and Bollerslev and Wooldrige (1992) method for standard errors estimates. GARCH model parameters are estimated using the quasi-maximum likelihood - QML (Brooks, 2008, p. 399). The maximum likelihood estimation produces asymptotically more efficient estimates than the estimates which can be obtained by using other methods.

Data and Empirical Results

The data used in this paper are the daily exchange rates of the euro against the dollar in the period from 03.01.2000 to 30.09.2016 (a total of 4291 pieces of data). We used the IMF data on the daily spot exchange rates: <https://www.imf.org/external/np/fin/ert/GUI/Pages/CountryDataBase.aspx> The nominal increase in the exchange rate means appreciation of the

Grafikon 1. Dnevna stopa prinosa evra prema dolaru



Izvor: Obrada autora.

U grafikonu 1. se uočava da dnevna stopa prinosa evra prema dolaru oscilira oko nulte vrednosti. Zapaža se prisustvo nekoliko nestandardnih opservacija, a oscilacije su pojačane tokom 2008. godine usled svetske ekonomske krize.

Tabela 1. Deskriptivna statistika dnevne stope prinosa evra

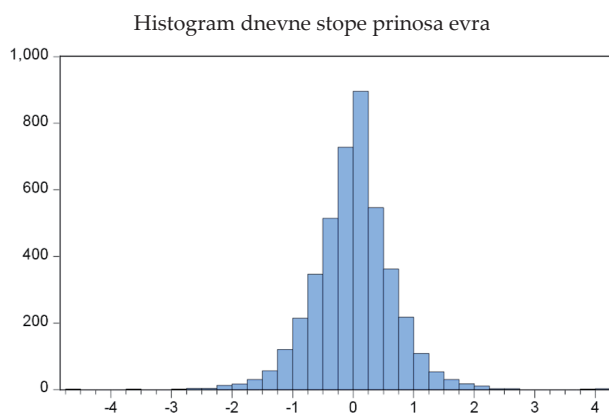
| | |
|---------------------------|-----------|
| Aritmetička sredina | 0,002351 |
| Medijana | 0,000000 |
| Maksimalna vrednost | 4,204134 |
| Minimalna vrednost | -4,735441 |
| Standardna devijacija | 0,643764 |
| Koeficijent asimetrije | -0,039528 |
| Koeficijent spljoštenosti | 5,850363 |
| Žark-Bera | 1453,722 |
| Verovatnoća p | 0,000000 |
| Zbir | 10,08807 |
| Zbir kvadrata odstupanja | 1777,913 |
| Broj posmatranja | 4291 |

Izvor: Obrada autora pomoću softverskog paketa EViews.

Osnovne deskriptivne statistike serije prinosa evra izložene su u tabeli 1. Srednja vrednost serije ne razlikuje se značajnije od nule. Koeficijent asimetrije je negativan (-0,039528), što znači da je serija asimetrična ulevo. Koeficijent spljoštenosti iznosi 5,850363. Vrednost ovog koeficijenta u slučaju normalne raspodele je tri. Dakle, pošto je koeficijent spljoštenosti veći od tri, repovi serije prinosa evra su teži od repova normalne raspodele. Ekstremne vrednosti u seriji dovode do formiranja

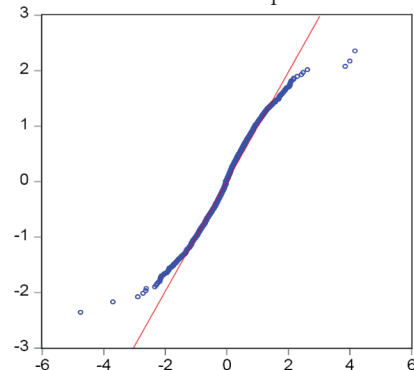
teških repova. Na osnovu p-vrednosti Žark-Bera (Jarque-Bera) test statistike, koja je nula do šeste decimale, odbacuje se nulta hipoteza o normalnost stope prinosa evra za nivo značajnosti 5% i prihvata alternativna po kojoj stope prinosa ne poseduje normalnu raspodelu. Na osnovu histograma i Q-Q dijagrama prinosa evra (Grafikon 2) takođe zaključujemo da empirijska raspodela serije prinosa odstupa od normalne raspodele.

Grafikon 2.



Izvor: Autor

Q-Q dijagram prinosa evra u odnosu na normalnu raspodelu



euro against the dollar. The baseline data were log transformed. Daily returns of exchange rate are calculated as $r_t = \log(y_t) - \log(y_{t-1}) \times 100$, where y_t is the level of the prompt exchange rate at the time t , wherein $t=1, 2, \dots, T$. *According to this approach, the daily euro appreciation or depreciation against the dollar was obtained as the first difference of the log exchange rate level.* The daily euro returns against the dollar are shown in Figure 1.

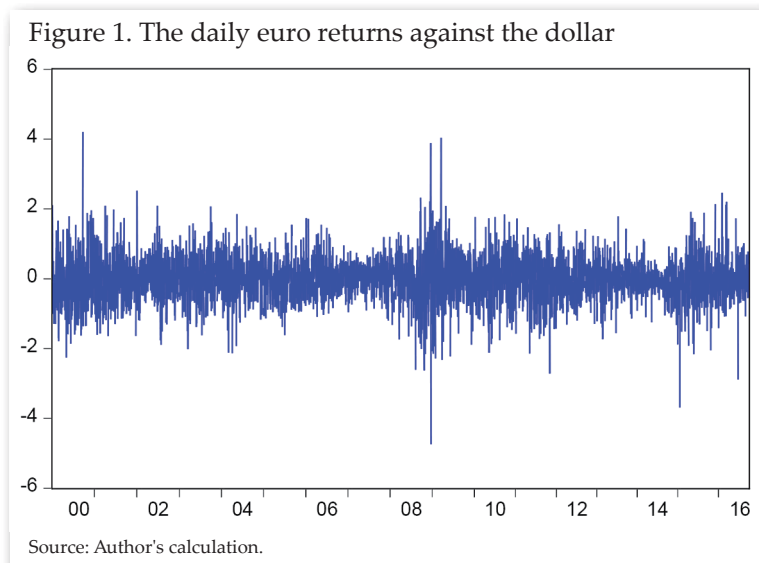


Figure 1 shows that the daily returns of the euro against the dollar fluctuate around zero. The presence of several non-standard observations can be seen, but fluctuations were intensified in 2008 due to the global economic crisis.

The basic descriptive statistics of the euro returns are set out in Table 1. The mean value of the series does not differ significantly from zero. The coefficient of skewness is -0.039528, which means that the series is skewed to the left. Kurtosis is 5.850363. The value of this ratio in the case of normal distribution is three. So, as the kurtosis is greater than three, the tails of the euro returns are heavier than the normal distribution. The extreme values in the series lead to the formation of heavy tails. Based on the p-value of the Jarque-Bera test statistic, which is zero to six decimal places, the null hypothesis of normality of the euro returns is rejected at the 5% level and the alternative one is accepted according to which the returns are not normally distributed. Based on the histogram and Q-Q graph of the euro returns - REUR (Figure 2) we also conclude that the empirical distribution of the euro returns differs from the normal distribution.

Table 1. Descriptive statistics of the daily euro returns

| | |
|--------------|-----------|
| Mean | 0.002351 |
| Median | 0.000000 |
| Maximum | 4.204134 |
| Minimum | -4.735441 |
| Std. Dev. | 0.643764 |
| Skewness | -0.039528 |
| Kurtosis | 5.850363 |
| Jarque-Bera | 1453.722 |
| Probability | 0.000000 |
| Sum | 10.08807 |
| Sum Sq. Dev. | 1777.913 |
| Observations | 4291 |

Source: Author by EViews software package.

Radi provere da li je serija dnevnih prinosa deviznog kursa evra prema dolaru stacionarna, sprovedi smo testiranje o postojanju jediničnog korena. Korišćena su dva testa jediničnog korena: Prošireni Diki-Fulerov (Augmented Dickey-Fuller - ADF) test i KPSS test (KPSS je skraćenica sastavljena od prvih slova prezimena autora Kwiatkowski-Phillips-Schmidt-Shin). Rezultati testiranja su izloženi u tabeli 2.

Analiza običnog i parcijalnog korelograma kvadrirane serije prinosa ukazuje na statističku značajnost većeg broja koeficijenata (grafikoni 3. i 4). Vrednost autokorelacionog koeficijenta prvog reda je 0,18 koja zatim oscilirajuće opada do vrednosti 0,04 na 33 docnji. Mada vrednosti autokorelacionih koeficijenata nisu visoke, one su statistički značajne. Dobijene p-vrednosti za ove koeficijente su nula do vrednosti četvrte

Tabela 2. ADF i KPSS test jediničnog korena

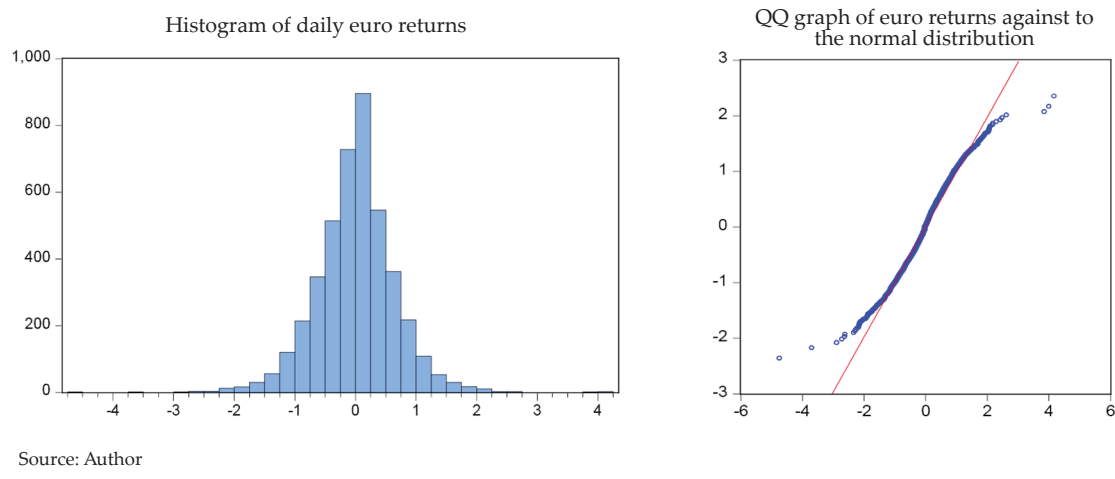
| Prošireni Diki-Fulerov test | | | | | |
|--|---|------------------------------------|--|----------------------------|----------------------------|
| $H_0: I(1), H_1: I(0)$ | | | | | |
| Variabla | Frekvencija | ADF test statistika (t-statistika) | Maksimalna docnja | Kritična vrednost, nivo 5% | Determinističke komponente |
| Stopa prinosa evra | 03.01.2000.- 30.09.2016. (dnevno) | -65,74 | 30 | -2,86 | Konstanta (C) |
| Napomene: Proširena Diki-Fulerova test statistika; Docnja: 0 (Automatski izbor zasnovan na SIC). | | | | | |
| KPSS test | | | | | |
| $H_0: I(0), H_1: I(1)$ | | | | | |
| Variabla | Frekvencija | KPSS test statistika (LM-stat.) | Asimptotska kritična vrednost, nivo 5% | Determinističke komponente | |
| Stopa prinosa evra | 03.01.2000.- 30.09.2016. (dnevna) | 0,208 | 0,463 | Konstanta (C) | |
| Napomene: Kwiatkowski-Phillips-Schmidt-Shin test statistika; Bandwidth: 5 (Newey-West automatski) koristeći Bartlett kernel. | | | | | |

Izvor: Obrada autora pomoću softverskog paketa EViews.

Na osnovu oba testa jediničnog korena, gde je kao deterministička komponenta korišćena samo konstanta, zaključuje se da je serija prinosa deviznog kursa evra prema dolaru stacionarna. Isti zaključak važi i za slučaj kad se kao deterministička komponenta uzmu konstanta i trend.

decimale, na osnovu čega se odbacuje nulta hipoteza o nepostojanju ARCH strukture. Parcijalni autokorelogram takođe skreće pažnju da postoji više parcijalnih autokorelacionih koeficijenata koji su statistički značajni. Na osnovu dobijenih ocena ovog koeficijenta može se takođe uočiti da vremenska serija prinosa poseduje autoregresionu strukturu varijabiliteta. Stoga se na osnovu običnog i parcijalnog korelograma kvadrata serije prinosa evra zaključuje da ova serija poseduje autoregresione karakteristike.

Figure 2.



To check whether the series of daily euro returns against the dollar is stationary, we have conducted a unit root test. We have used the two unit root test: Augmented Dickey-Fuller - ADF test and the KPSS test (KPSS is an acronym composed of the first letters of the authors' surnames Kwiatkowski-Phillips-Schmidt-Shin). The test results are set out in Table 2.

The analysis of ordinary and partial correlogram of squared returns indicates that several coefficients are statistically significant (Charts 3 and 4). The first order autocorrelation coefficient value is 0.18, which then oscillates down to the values of 0.04 with 33 lags. Although the value of autocorrelation coefficients is not high, they are statistically significant. The

Table 2. ADF and KPSS unit root test

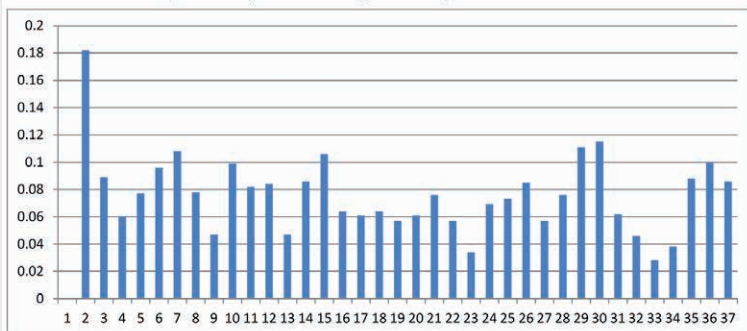
| ADF Unit Root Test | | | | | |
|--|-------------------------------------|----------------------------------|--------------------------------------|--------------------------------|--------------------------|
| | | $H_0: I(1), H_1: I(0)$ | | | |
| Variable | Frequency | ADF test statistic (t-statistic) | Maxlag | Test critical values, 5% level | Deterministic components |
| Euro returns | 03.01.2000 30.09.2016 (daily) | -65,74 | 30 | -2,86 | Constant (C) |
| Note: Augmented Dickey-Fuller test statistic; Lag Length: 0 (Automatic-based on SIC). | | | | | |
| KPSS Unit Root Test | | | | | |
| | | $H_0: I(0), H_1: I(1)$ | | | |
| Variable | Frequency | KPSS test statistic (LM-stat.) | Asymptotic critical values, 5% level | | Deterministic components |
| Euro returns | 03.01.2000 30.09.2016 (daily) | 0,208 | 0,463 | | Constant (C) |
| Note: Kwiatkowski-Phillips-Schmidt-Shin test statistic; Bandwidth: 5 (Newey-West automatic) using Bartlett kernel. | | | | | |

Source: Author by EViews software package.

According to the both unit root tests, where the only deterministic component is the constant term, it can be concluded that the series of the euro returns against the dollar is stationary. The same conclusion applies if the deterministic components are the constant term and the trend.

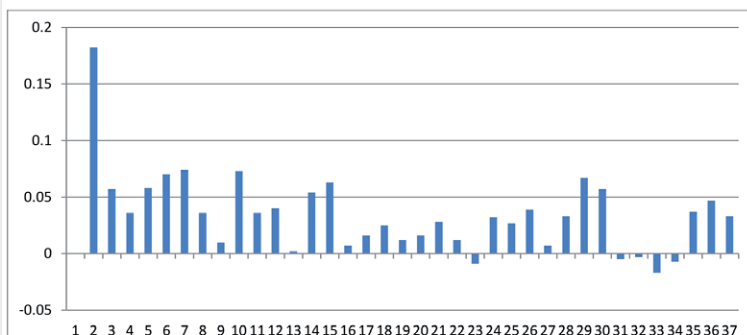
realized p-values for these coefficients are zero to four decimals, based on which the null hypothesis of no ARCH structure is rejected. The partial autocorrelations also highlight that there are more partial autocorrelation coefficients which are statistically significant. According to the estimated coefficient it can

Grafikon 3. Autokorelacija kvadrata prinosa deviznog kursa evra prema dolaru



Izvor: Autor

Grafikon 4. Parcijalna autokorelacija kvadrata prinosa deviznog kursa evra prema dolaru



Izvor: Autor

Na osnovu visokih vrednosti Q^2 i ARCH statistike zaključujemo da serija prinosa deviznog kursa evra poseduje nestabilnu uslovnu varijansu. To zapravo znači da se u modeliranju može primeniti familija ARCH modela. ARCH model reda 5 poseduje zadovoljavajuća statistička svojstva. Pošto u vremenskoj seriji prinosa evra postoji nekoliko nestandardnih opservacija, oceničeno ARCH model reda 5 uvodeći u jednačinu srednje vrednosti veštačke promenljive za sledeće datume 22/09/2000, 18/12/2008, 19/12/2008, 19/03/2009, 01/11/2011, 23/01/2015, 04/02/2016 i 24/06/2016. Veštačke promenljive su označene od D1 do D8, respektivno. Time se modeliraju najizraženije oscilacije (tabela 4).

U postupku provere postojanja nestabilne uslovne varijanse prvi korak je da se oceni linearni model, posle čega će se izvršiti testiranje reziduala na postojanje ARCH strukture. Serija prinosa je modelirana u funkciji ARMA (1,1) koja je izabrana arbitrarno u ovoj fazi modeliranja. Postojanje nestabilne varijanse u vremenskoj seriji prinosa evra proverava se pomoću Ljung-Boksove (Ljung-Box) statistike (Q^2) i Engleove ARCH statistike. Vrednosti u tabeli 3. dobijene su iz reziduala regresije kojom je serija prinosa evra modelirana kao funkcija konstante i ARMA(1,1).

Tabla 3. Testovi postojanja ARCH efekta u seriji prinosa evra

| Std. dev. | Asimetrija | Spljoštenost | JB | $Q^2(5)$ | $Q^2(10)$ | $Q^2(20)$ | ARCH $\chi^2(5)$ | ARCH $\chi^2(10)$ | ARCH $\chi^2(20)$ |
|-------------|------------|--------------|-----------------|-------------------|-------------------|-------------------|------------------|-------------------|-------------------|
| 0,643 | -0,042 | 5,848 | 1450 (0,00)* | 259,60 (0,00)* | 417,15 (0,00)* | 647,24 (0,00)* | 197,3 (0,00)* | 251,8 (0,00)* | 294,2 (0,00)* |
| *p vrednost | | | | | | | | | |

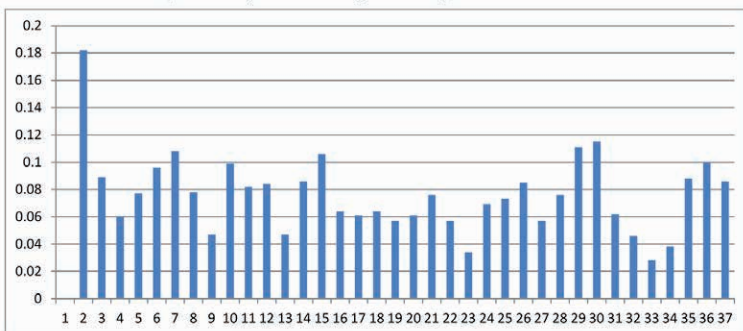
Izvor: Obrada autora pomoću softverskog paketa EViews.

also be seen that the time series of returns has the autoregression structure of variability. Therefore, based on the common and partial correlogram of squared daily returns it can be concluded that this series is autoregressive.

In the process of checking the existence of unstable conditional variance the first step is to assess the linear model, after which the testing of residuals to the existence of ARCH structure will be performed. The series of returns is

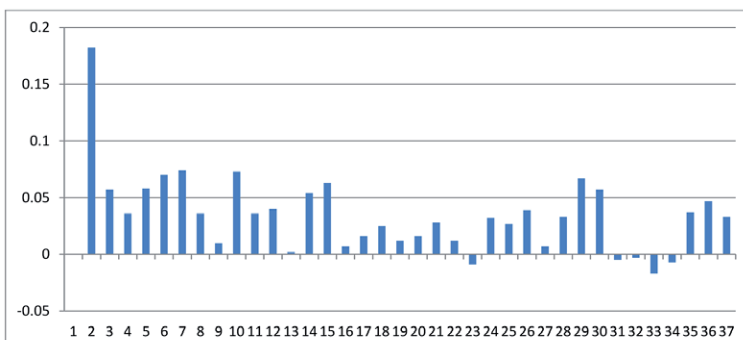
modeled as a function of ARMA (1,1), which is arbitrarily selected at this stage of the modeling. The existence of unstable variance in the time series of euro returns is checked using the Ljung-Box statistics (Q2) and Engle ARCH statistics. The values in Table 3 were obtained from the residuals of the regression which is a series of euro returns modeled as a function of the constants and ARMA (1,1).

Figure 3. Autocorrelation of the euro squared returns against the dollar



Source: Author.

Figure 4. Partial autocorrelation of squared returns of the euro exchange rate against the dollar



Source: Author.

Table 3. Testing the existence of ARCH effect in a series of the euro returns

| Std. Dev. | Skewness | Kurtosis | JB | Q ² (5) | Q ² (10) | Q ² (20) | ARCH χ^2 (5) | ARCH χ^2 (10) | ARCH χ^2 (20) |
|-----------|----------|----------|-----------------|--------------------|---------------------|---------------------|-------------------|--------------------|--------------------|
| 0,643 | -0,042 | 5,848 | 1450 (0,00)* | 259,60 (0,00)* | 417,15 (0,00)* | 647,24 (0,00)* | 197,3 (0,00)* | 251,8 (0,00)* | 294,2 (0,00)* |
| *p value | | | | | | | | | |

Source: Author by EViews software package.

Tabela 4. ARCH(5) sa veštačkim promenljivima u jednačini srednje vrednosti

| Jednačina srednje vrednosti | | | | |
|--|-----------|-------------------|--------------|------------|
| Promenljiva | Ocena | Standardna greška | z-statistika | p-vrednost |
| Konstanta | 0,010837 | 0,008930 | 1,213515 | 0,2249 |
| D1 | 4,191181 | 0,009199 | 455,6199 | 0,0000 |
| D2 | 3,874446 | 0,008931 | 433,8233 | 0,0000 |
| D3 | -4,601231 | 0,483530 | -9,515924 | 0,0000 |
| D4 | 4,021212 | 0,010615 | 378,8114 | 0,0000 |
| D5 | -2,698825 | 0,021680 | -124,4828 | 0,0000 |
| D6 | -3,714043 | 0,023157 | -160,3849 | 0,0000 |
| D7 | 2,452770 | 0,009397 | 261,0143 | 0,0000 |
| D8 | -2,889305 | 0,009038 | -319,6841 | 0,0000 |
| Jednačina volatilnosti | | | | |
| Promenljiva | Ocena | Standardna greška | z-statistika | p-vrednost |
| Konstanta | 0,259358 | 0,015410 | 16,83027 | 0,0000 |
| ARCH (1) | 0,033163 | 0,016661 | 1,990500 | 0,0465 |
| ARCH (2) | 0,062144 | 0,017859 | 3,479740 | 0,0005 |
| ARCH (3) | 0,073007 | 0,018472 | 3,952349 | 0,0001 |
| ARCH (4) | 0,077132 | 0,019721 | 3,911232 | 0,0001 |
| ARCH (5) | 0,070398 | 0,018033 | 3,903897 | 0,0001 |
| Q(10)=4,74(0,91), Q(20)=12,39(0,90), Q(30)=22,73(0,83), Q ² (10)=37,99(0,00), Q ² (20)=138,46(0,00), Q ² (30)=246,37(0,00), ARCH 10=38,93(0,00), ARCH(20)=133,23(0,00), ARCH(30)=193,41(0,00) Koeficijent asimetrije = -0,04 Koeficijent spljoštenosti = 3,94 JB=158,33 | | | | |

Napomene: U zagradi uz koeficijente u donjem delu tabele data je p-vrednost. Korišćen je ARCH χ^2 test. Ove napomene se odnose na tabele br. 4, 5. i 6.

Izvor: Obrada autora pomoću softverskog paketa EViews.

Ocenjeni koeficijenti u modelu ARCH(5) su statistički signifikantni. Standardizovani reziduali u ocenjenom modelu ARCH(5) nisu autokorelisani. Odsustvo korelacije zapaža se i kod kvadriranih standardizovanih reziduala do koeficijenta sa rednim brojem šest. Model u značajnoj meri obuhvata dinamiku nestabilne uslovne varijanse. Za ARCH test važi sledeća konstatacija: Ako je vrednost test statistike veća od kritične vrednosti χ^2 distribucije, odbacuje se nulta hipoteza o odsustvu autokorelacije. U ocenjenom modelu ARCH(5) vrednost JB

statistike je manja nego kod originalne serije prinosa evra. Verovatnoća p uz JB statistiku je nula do vrednosti četvrte decimale, što vodi odbacivanju nulte hipoteze o normalnoj distribuciji. Odstupanje od normalne raspodele se duuguje povećanoj vrednosti koeficijenta spljoštenosti. Sve dobijene vrednosti nedvosmisleno ukazuju na prisustvo nestabilne varijanse, a time i ARCH strukture. To znači da se može primeniti familija ARCH modela za modeliranje serije prinosa.

On the basis of high Q^2 values and ARCH statistics we conclude that the series of euro-dollar exchange rate returns has an unstable conditional variance. This actually means that in the modeling of the series of euro returns, the family of ARCH models can be applied. The ARCH model of the order of 5 possesses the satisfactory statistical properties. Given that the time series of euro returns includes several non-standard observations, we will estimate the ARCH model of the order of 5 by introducing into the mean equation the dummy variables for the following dates: 22/09/2000, 18/12/2008, 19/12/2008, 19/03/2009, 01/11/2011, 23/01/2015, 04/02/2016 and 24/06/2016. The dummy variables are labeled from D1 to D8, respectively. Thus, the most pronounced fluctuations are modeled (Table 4).

of a correlation can be seen in respect of the squared standardized residuals up to the coefficient with the ordinal number six. The model substantially covers the dynamics of unstable conditional variance. For the ARCH test, the following statement applies: If the value of the test statistic exceeds the critical value of χ^2 distribution, the null hypothesis about the absence of autocorrelation is rejected. In the estimated model ARCH (5) JB statistic is less than the original return series. The probability p with Jarque-Bera (JB) statistic is zero to four decimal values, leading to the rejection of the null hypothesis of normal distribution. The deviations from the normal distribution are owing to the increased coefficient of skewness. All obtained values clearly indicate the presence of an unstable variance and thus the ARCH

Table 4. ARCH(5) with dummy variables in the mean equation

| The mean equation | | | | |
|---|-------------|----------------|-------------|--------|
| Variable | Coefficient | Standard Error | z-Statistic | Prob. |
| Constant | 0,010837 | 0,008930 | 1,213515 | 0,2249 |
| D1 | 4,191181 | 0,009199 | 455,6199 | 0,0000 |
| D2 | 3,874446 | 0,008931 | 433,8233 | 0,0000 |
| D3 | -4,601231 | 0,483530 | -9,515924 | 0,0000 |
| D4 | 4,021212 | 0,010615 | 378,8114 | 0,0000 |
| D5 | -2,698825 | 0,021680 | -124,4828 | 0,0000 |
| D6 | -3,714043 | 0,023157 | -160,3849 | 0,0000 |
| D7 | 2,452770 | 0,009397 | 261,0143 | 0,0000 |
| D8 | -2,889305 | 0,009038 | -319,6841 | 0,0000 |
| Variance equation | | | | |
| Variable | Coefficient | Standard Error | z-Statistic | Prob. |
| Constant | 0,259358 | 0,015410 | 16,83027 | 0,0000 |
| ARCH (1) | 0,033163 | 0,016661 | 1,990500 | 0,0465 |
| ARCH (2) | 0,062144 | 0,017859 | 3,479740 | 0,0005 |
| ARCH (3) | 0,073007 | 0,018472 | 3,952349 | 0,0001 |
| ARCH (4) | 0,077132 | 0,019721 | 3,911232 | 0,0001 |
| ARCH (5) | 0,070398 | 0,018033 | 3,903897 | 0,0001 |
| Q(10)=4,74(0,91), Q(20)=12,39(0,90), Q(30)=22,73(0,83), Q ² (10)=37,99(0,00), Q ² (20)=138,46(0,00), Q ² (30)=246,37(0,00), ARCH 10=38,93(0,00), ARCH(20)=133,23(0,00), ARCH(30)=193,41(0,00) Skewness = -0,04 Kurtosis = 3,94 JB=158,33 | | | | |

Note: The numbers in parentheses next to the coefficients in the lower part of the table are p-values. ARCH χ^2 test was applied. These notes refer also to Tables 4, 5 and 6.
Source: Author by EViews software package.

The estimated coefficients in the model ARCH (5) are statistically significant. Standardized residuals in the estimated model ARCH (5) are not autocorrelated. The absence

structure. This means that the family of ARCH models can be used for modeling a series of returns.

Tabela 5. Ocene parametara GARCH (1,1) modela sa veštačkim promenljivima u jednačini srednje vrednosti

| Jednačina srednje vrednosti | | | | |
|--|-----------|------------|--------------|------------|
| Promenljiva | Ocena | St. greška | z-statistika | p-vrednost |
| C-Konstanta | 0,007488 | 0,008405 | 0,890929 | 0,3730 |
| D1 | 4,196646 | 0,008405 | 499,3267 | 0,0000 |
| D2 | 3,877918 | 0,008405 | 461,4038 | 0,0000 |
| D3 | -4,742930 | 0,008405 | -564,3249 | 0,0000 |
| D4 | 4,030223 | 0,008405 | 479,5254 | 0,0000 |
| D5 | -2,715051 | 0,008405 | -323,0432 | 0,0000 |
| D6 | -3,689531 | 0,008405 | -438,9890 | 0,0000 |
| D7 | 2,458873 | 0,008405 | 292,5626 | 0,0000 |
| D8 | -2,892006 | 0,011295 | -256,0396 | 0,0000 |
| Jednačina volatilnosti | | | | |
| Promenljiva | Ocena | St. greška | z-statistika | p-vrednost |
| C-Konstanta | 0,001190 | 0,000510 | 2,331427 | 0,0197 |
| ARCH(1) | 0,028157 | 0,004686 | 6,008339 | 0,0000 |
| GARCH(1) | 0,968621 | 0,005204 | 186,1250 | 0,0000 |
| Q(10)=5,94(0,82), Q(20)=13,97(0,83), Q ² (10)=7,32(0,70), Q ² (20)=19,53(0,43), ARCH10=7,52(0,68), ARCH20=19,20(0,46), JB=103,25. | | | | |

Izvor: Obrada autora pomoću softverskog paketa EViews.

Glavna teškoća kod primene ARCH(q) modela je određivanje reda docnje q. Ovaj problem se može delimično rešiti pomoću testa količnika verovatnoće. Zapravo, zahtevani red docnje kvadrata grešaka da bi se obuhvatile sve zavisnosti u jednačini uslovne varijanse može da bude veliki. U cilju prevazilaženja ovih problema, u literaturi su konstruisani GARCH modeli. Da bi se ocenili modeli familije GARCH, koristi se metod maksimalne verodostojnosti. U ovom radu je primenjen Marquardt algoritam numeričke optimizacije, koji ujedno predstavlja modifikaciju BHHH algoritma (oba algoritma su varijante Gaus-Njutnovog metoda). Marquardt algoritam poseduje snagu "korekcije" koja brže potiskuje ocenjene koeficijente prema njihovoj optimalnoj vrednosti (Videti detaljnije kod Press et. al., 1992).

U tabeli 5. su ocenjeni parametri GARCH(1,1) modela. U jednačinu srednje vrednosti su uključene veštačke promenljive obuhvaćene prethodnom ocenom ARCH(5) modela. Ocenjeni koeficijent kvadriranih reziduala na docnji ((ARCH) i koeficijent uz parametar uslovne varijanse (GARCH) u jednačini uslovne varijanse su naglašeno statistički značajni. Zbir vrednosti ocene ova dva koeficijenta je blizu jedinice (što je tipično za ocenjene GARCH modele za podatke prinosa finansijske aktive). To znači da će šokovi u jednačini uslovne

varijanse biti dugotrajni. Velika vrednost zbira ova dva koeficijenta sugerise da visoka stopa pozitivnog ili negativnog prinosa dovodi do velikih prognoziranih vrednosti varijanse u produženom periodu. Pojedinačni koeficijenti uslovne varijanse su u skladu sa očekivanjem. Koeficijent odsečka "C" je veoma mali, ARCH parametar je oko 0,03 (ocenjen koeficijent α_1), dok je koeficijent uslovne varijanse na docnji ("GARCH") veoma naglašen (0,97) (ocenjen koeficijent β_1). Uslov stabilnosti GARCH modela je $\alpha_1 + \beta_1 < 1$. α_1 je parametar koji određuje koliko snažno promena prinosa utiče na volatilnost. β_1 - varijansa iz prethodnog perioda - parametar koji određuje promenu volatilnosti u vremenu. Uz ograničenje $\beta_1 = \beta_2 = \dots = \beta_s = 0$, GARCH(m,s) specifikacija se svodi na model autoregresione uslovne heteroskedastičnosti, u oznaci: ARCH(m). U poslednjoj koloni u tabeli 5. su prikazane p-vrednosti.

Dakle, tri ocenjena koeficijenta u jednačini varijanse u tabeli 5. predstavljaju sledeće veličine: C - konstantu jednačine, ARCH(1) - koeficijent ispred kvadrata reziduala na prvoj docnji i GARCH(1) - koeficijent ispred uslovne varijanse, takođe na prvoj docnji (Engle, 2001, str. 163). Za stabilnost varijanse potrebno je da zbir ocenjenih koeficijenata α_1 i β_1 bude manji od 1. U tabeli 5. ovaj uslov je ispunjen. U ovoj tabeli su date vrednosti standardnih grešaka

Table 5. Parameter estimates of the GARCH (1,1) model with dummy variables in the mean equation

| The mean equation | | | | |
|--|-------------|----------------|-------------|--------|
| Variable | Coefficient | Standard Error | z-Statistic | Prob. |
| C-Constant | 0,007488 | 0,008405 | 0,890929 | 0,3730 |
| D1 | 4,196646 | 0,008405 | 499,3267 | 0,0000 |
| D2 | 3,877918 | 0,008405 | 461,4038 | 0,0000 |
| D3 | -4,742930 | 0,008405 | -564,3249 | 0,0000 |
| D4 | 4,030223 | 0,008405 | 479,5254 | 0,0000 |
| D5 | -2,715051 | 0,008405 | -323,0432 | 0,0000 |
| D6 | -3,689531 | 0,008405 | -438,9890 | 0,0000 |
| D7 | 2,458873 | 0,008405 | 292,5626 | 0,0000 |
| D8 | -2,892006 | 0,011295 | -256,0396 | 0,0000 |
| Variance equation | | | | |
| Variable | Coefficient | Standard Error | z-Statistic | Prob. |
| C-Constant | 0,001190 | 0,000510 | 2,331427 | 0,0197 |
| ARCH(1) | 0,028157 | 0,004686 | 6,008339 | 0,0000 |
| GARCH(1) | 0,968621 | 0,005204 | 186,1250 | 0,0000 |
| Q(10)=5,94(0,82), Q(20)=13,97(0,83), Q ² (10)=7,32(0,70), Q ² (20)=19,53(0,43), ARCH10=7,52(0,68), ARCH20=19,20(0,46), JB=103,25. | | | | |

Source: Author by EViews software package.

The main difficulty in the application of the ARCH (q) model is to determine the order of delay q. This problem can be partially solved by using the probability ratio test. In fact, in order to include all dependencies in the equation of conditional variance, the required order of delay of squared errors can be huge. In order to overcome these problems, GARCH models were developed in the reference literature. With a view to assessing the GARCH family models, the maximum likelihood method is used. In this paper we used the Marquardt algorithm numerical optimization, which also represents a modification of the BHHH algorithm (both algorithms are variants of the Gauss-Newton method). The Marquardt algorithm possesses the power of "correction" that quickly pushes up the estimated coefficients to their optimum values (for more details see Press et al., 1992).

The GARCH (1,1) model parameter estimates are presented in table 5. The dummy variables are included in the mean equation for the estimation of the ARCH (5) model. The estimated coefficient of the lagged squared residuals ((ARCH) and the coefficient in the conditional variance equation (GARCH) are highly statistically significant. The sum of these two estimates is close to a unit (which is typical for the estimated GARCH models for financial assets returns). This means that the shocks in

the conditional variance equation will exhibit a long memory. The high value of the sum of these two coefficients suggests that the high rates of positive or negative returns lead to a large forecast of the variance value for the extended period. The individual coefficients of the conditional variance are in line with the expectations. The coefficient of constant "C" is very small, the ARCH parameter is about 0.03 (estimated coefficient α_1), while the coefficient of lagged conditional variance ("GARCH") is very pronounced (0.97) (estimated coefficient β_1). The condition of stability of the GARCH model is $\alpha_1 + \beta_1 < 1$. Parameter α_1 determines how strongly the changes in returns affect the volatility. The parameter β_1 - variance from the previous period - is the parameter that determines the change in volatility over time. With the restriction $\beta_1 = \beta_2 = \dots = \beta_s = 0$, GARCH(m,s) specification comes down to the autoregressive conditional heteroscedasticity ARCH (m) model. The last column of Table 5 shows the p-value.

Thus, the three estimated coefficients in the variance equation in Table 5 represent the following variables: C - intercept, ARCH (1) - the first lag of the squared residuals and GARCH (1) - the coefficient of the conditional variance, also with the first lag (Engle 2001, p. 163). For the purpose of variance stability,

i vrednosti z-statistike (količnici koeficijena i odgovarajućih standardnih grešaka) i p-vrednosti. Dobijeni koeficijenti su statistički signifikantni, sem vrednosti konstante u jednačini srednje vrednosti. GARCH proces karakteriše stabilna srednja vrednost i uslovna heteroskedastičnost, pri čemu je neuslovna varijansa konstantna. U ocenjivanju jednačine GARCH specifikacije primenjen je metod maksimalne verodostojnosti. Standardne greške i kovarijansa izračunate su robustnim metodom Bollerslev-Woodridge-a.

Statistička svojstva GARCH(1,1) modela su prihvatljiva. Podsetimo se da se Q statistika odnosi na standardizovane rezidualne i da se koristi za testiranje postojanja preostale serijske korelacije u jednačini srednje vrednosti i proveru specifikacije ove jednačine. Ako je jednačina srednje vrednosti korektno specifikovana, ni jedna Q statistika neće

biti signifikantna. Korelogram kvadriranih standardizovanih reziduala koristi se za testiranje preostalog ARCH efekta u jednačini uslovne varijanse i proveru specifikacije ove jednačine ((Standardizovani reziduali se dobijaju tako što se serija reziduala podeli sa odgovarajućim ocenama standardne devijacije ($\frac{\hat{\epsilon}_t}{\hat{\sigma}_t}$)). Autokorelacija kvadrata standardizovanih reziduala ocenjenog modela GARCH(1,1) je znatno manja u poređenju sa autokorelacionom strukturom prinosa deviznog kursa evra. Izračunate p-vrednosti uz ocenjene vrednosti kvadrata standardizovanih reziduala su iznad 0,05 zbog čega možemo prihvatiti hipotezu da reziduali ne poseduju ARCH strukturu.

U nameri da proverimo u kojoj meri se eksponencijalni GARCH model može prilagoditi podacima vremenske serije prinosa deviznog kursa evra, ocenili smo ovaj model (tabela 6).

Tabela 6. Ocene parametara asimetričnog EGARCH (1,1) modela sa veštačkim promenljivima u jednačini srednje vrednosti

| Jednačina srednje vrednosti | | | | |
|--|-----------|------------|-----------|------------|
| Promenljiva | Oцена | St. greška | z-odnos | p-vrednost |
| C-Konstanta | 0,004817 | 0,008501 | 0,566614 | 0,5710 |
| D1 | 4,244154 | 0,147878 | 28,70032 | 0,0000 |
| D2 | 4,098139 | 0,311360 | 13,16206 | 0,0000 |
| D3 | -4,484761 | 0,334735 | -13,39795 | 0,0000 |
| D4 | 4,098440 | 0,255377 | 16,04862 | 0,0000 |
| D5 | -2,713536 | 0,111725 | -24,28756 | 0,0000 |
| D6 | -3,609826 | 0,163543 | -22,07266 | 0,0000 |
| D7 | 2,305701 | 0,198016 | 11,64404 | 0,0000 |
| D8 | -2,865515 | 0,089591 | -31,98438 | 0,0000 |
| Jednačina volatilnosti | | | | |
| Promenljiva | Oцена | St. greška | z-odnos | p-vrednost |
| C-Konstanta | -0,062317 | 0,010862 | -5,737401 | 0,0000 |
| α ARCH | 0,072323 | 0,012601 | 5,739323 | 0,0000 |
| γ Leveridž efekat - RESID(-1)/@SQRT(GARCH(-1)) | -0,009367 | 0,006802 | -1,377021 | 0,1685 |
| β GARCH | 0,993729 | 0,001995 | 498,1575 | 0,0000 |
| Q(10)=6,10(0,81), Q(20)=14,06(0,83), Q ² (10)=9,47(0,49), Q ² (20)=21,31(0,38), ARCH10=9,78(0,46), ARCH20=21,76(0,35), Koef. asimetrije = -0,05 Koef. spljoštenosti = 3,75 JB=102,14 (0,000); U zagradama je p-vrednost. Za ARCH je data hi kvadrat verovatnoća. | | | | |

Izvor: Obrada autora pomoću softverskog paketa EViews.

the sum of the estimated coefficients α_1 and β_1 should be less than 1. In Table 5 this condition is met. This table features the values of standard errors, z-statistics (ratio between coefficient and the corresponding standard error), and the p-value. The obtained coefficients are statistically significant, except for the value of the constants in the mean equation. The GARCH process is characterized by the stable mean value and conditional heteroskedasticity, while nonconditional variance is constant. To estimate the equation of GARCH specification, the maximum likelihood method was applied. Standard errors and covariance were calculated by means of the robust Bollerslev-Woodridge method.

The statistical properties of the GARCH (1,1) model are acceptable. Let us recall that the Q statistic refers to the standardized residuals and is used to test the existence of residual serial correlation in the mean equation and

check the specifications of this equation. If the mean equation is correctly specified, Q statistics will not be significant. The correlogram of the squared standardized residuals is used to test the remaining ARCH effect in the conditional variance equation and check the specifications of the equation (the standardized residuals are the residuals divided by the relevant assessments of standard deviation ($\frac{\hat{\epsilon}_t}{\hat{\sigma}_t}$)). Autocorrelation of squared standardized residuals in the estimated GARCH model (1.1) is significantly lower than the autocorrelation structure of the euro exchange rate returns. The calculated p-values with estimated value of the squares standardized residuals are above 0.05 due to which we can accept the hypothesis that the residuals do not have the ARCH structure.

In order to check to what extent the exponential GARCH model can be adapted to the data of the euro exchange rate return series, we have estimated this model (Table 6).

Table 6. The parameter estimates of the asymmetric EGARCH (1,1) model with dummy variables in the mean equation

| Mean equation | | | | |
|---|-------------|------------|-------------|--------|
| Variable | Coefficient | Std. Error | z-statistic | Prob. |
| Constant-C | 0,004817 | 0,008501 | 0,566614 | 0,5710 |
| D1 | 4,244154 | 0,147878 | 28,70032 | 0,0000 |
| D2 | 4,098139 | 0,311360 | 13,16206 | 0,0000 |
| D3 | -4,484761 | 0,334735 | -13,39795 | 0,0000 |
| D4 | 4,098440 | 0,255377 | 16,04862 | 0,0000 |
| D5 | -2,713536 | 0,111725 | -24,28756 | 0,0000 |
| D6 | -3,609826 | 0,163543 | -22,07266 | 0,0000 |
| D7 | 2,305701 | 0,198016 | 11,64404 | 0,0000 |
| D8 | -2,865515 | 0,089591 | -31,98438 | 0,0000 |
| Variance equation | | | | |
| Variable | Coefficient | Std. Error | z-statistic | Prob. |
| Constant-C | -0,062317 | 0,010862 | -5,737401 | 0,0000 |
| α ARCH | 0,072323 | 0,012601 | 5,739323 | 0,0000 |
| γ Leverage effect - RESID(-1)/@SQRT(GARCH(-1)) | -0,009367 | 0,006802 | -1,377021 | 0,1685 |
| β GARCH | 0,993729 | 0,001995 | 498,1575 | 0,0000 |
| Q(10)=6,10(0,81), Q(20)=14,06(0,83), Q ² (10)=9,47(0,49), Q ² (20)=21,31(0,38), ARCH10=9,78(0,46), ARCH20=21,76(0,35), Skewness = -0,05 Kurtosis = 3,75 JB=102,14 (0,000); In parentheses is the p-value. For ARCH chi-square probability is given. | | | | |

Source: Author by EViews software package.

Ocenjeni koeficijenti EGARCH(1,1) modela u jednačini srednje vrednosti su statistički signifikantni (sem konstante) (tabela 6). Leveridž efekat je pojava kad postoji korelacija između prošlih prinosa i buduće volatilnosti. Koeficijent uz leveridž efekat označićemo sa γ . U slučaju da je $\gamma < 0$, pozitivne (dobre) vesti sa tržišta generišu manju volatilnost u budućem periodu nego negativni šokovi. Koeficijent asimetrije $\text{RESID}(-1)/\sqrt{\text{GARCH}(-1)}$ u jednačini volatilnosti u tabeli 6. je negativan, i nije statistički signifikantan. Pouzdatnost dobijenih ocena EGARCH(1,1) modela ćemo proveriti pomoću nekoliko testova specifikacije, koji treba da potvde koliko je ocenjeni model saglasan sa vremenskom serijom prinosa evra.

Prema tabeli 6. uočava se da su standardizovani reziduali neautokorelisani (Ljung-Boksova test statistika - Q). Na taj zaključak navode visoke p-vrednosti uz ocenjene vrednosti Q statistike (na svim docnjama $p > 0,05$), što znači da se ne može odbaciti nulta hipoteza koja glasi: Nema autokorelacije do reda k . Pošto se nulta hipoteza ne može odbaciti, zaključujemo da nema zaostale autokorelacije u rezidualima ocenjenog modela. Korelogram kvadriranih standardizovanih reziduala (Q^2 statistika) takođe pokazuje da kvadrirani reziduali nisu autokorelisani, sugerišući da EGARCH model na statistički zadovoljavajući način obuhvata dinamiku uslovne varijanse. Koeficijenti autokorelacije (AC) i parcijalne autokorelacije (PAC) u korelogramu standardizovanih kvadriranih reziduala su na svim docnjama blizu nuli, tako da se može zaključiti da u rezidualima nema autoregresione uslovne heteroskedastičnosti na nivou značajnosti 0,05.

Postojanje ARCH efekta može se proveriti i preko ARCH statistike (Engle-ova LM test statistika). Ova statistika je asimptotski distribuirana kao Hi kvadrat raspodela ukoliko se ne može odbaciti nulta hipoteza po kojoj u rezidualima nema heteroskedastičnosti

odgovarajućeg reda. ARCH statistika se dobija kao proizvod koeficijenta determinacije iz modela u kome su kvadrirani podaci ocenjeni u odnosu na odgovarajući broj svojih prethodnih vrednosti i obima uzorka. Izračunate p-vrednosti u tabeli 6. uz ARCH statistiku na svim docnjama su veće od 0,05, tako da se ne može odbaciti nulta hipoteza o odsustvu ARCH efekta u rezidualima. Zbir ocenjenih parametara α i β je veći od jedan, što ukazuje na dugotrajan uticaj šokova na volatilnost.

Pošto je izračunata JB statistika u tabeli 6. veća od kritične vrednosti, a pri tome je p-vrednost do četvrte decimale nula, odbacuje se nulta hipoteza o postojanju normalne raspodele reziduala na nivou značajnosti 0,05 (Potrebno je istaći da se kvalitet modela ne dovodi u pitanje zbog odstupanja empirijske od normalne raspodele, jer nije nužno da slučajna greška poseduje normalnu raspodelu). Takođe QQ grafikon, koji predstavlja skup parova kvantila raspodele same serije i kvantila normalne raspodele, pokazuje da postoji neslaganje na krajevima (repovima) serije između empirijske i normalne raspodele. Vidimo da u grafikonu 5. postoje kako pozitivni tako i negativni šokovi, koji uslovljavaju odstupanje empirijske raspodele reziduala od normalne raspodele. Ranije smo istakli da su repovi empirijske raspodele prinosa finansijskih instrumenata često teži od repova normalne raspodele (Empirijska serija standardizovanih reziduala je normalno raspodeljena kad je njen treći momenat jednak nuli, a četvrti momenat jednak vrednosti 3). Mada se odgovarajući parovi kvantila na krajevima (veće vrednosti šokova) ne poklapaju sa kvantilima normalne raspodele, u velikom delu serije empirijska raspodela reziduala se može aproksimirati normalnom raspodelom. S obzirom na malu vrednost koeficijenta asimetrije (-0,05), može se reći da je empirijska raspodela simetrična. Međutim, zbog veće vrednosti koeficijenta spljoštenosti od vrednosti tri, dolazi do odstupanja od normalnosti.

The estimated coefficients of EGARCH (1,1) model in the mean equation are statistically significant (except the constant) (Table 6). The leverage effect is a phenomenon when there is a correlation between the past returns and future volatility. The coefficient of the leverage effect will be marked with γ . In the case of $\gamma < 0$, the positive (good) news from the market generate less volatility in the future than the negative shocks. The coefficient of skewness $\text{RESID}(-1)/\text{SQRT}((\text{GARCH}(-1)))$ in the volatility equation in Table 6 is negative and statistically significant. The reliability of the obtained estimates of the EGARCH(1,1) model will be verified using a series of test specifications, which should confirm how much the estimated model agrees with the time series of euro returns.

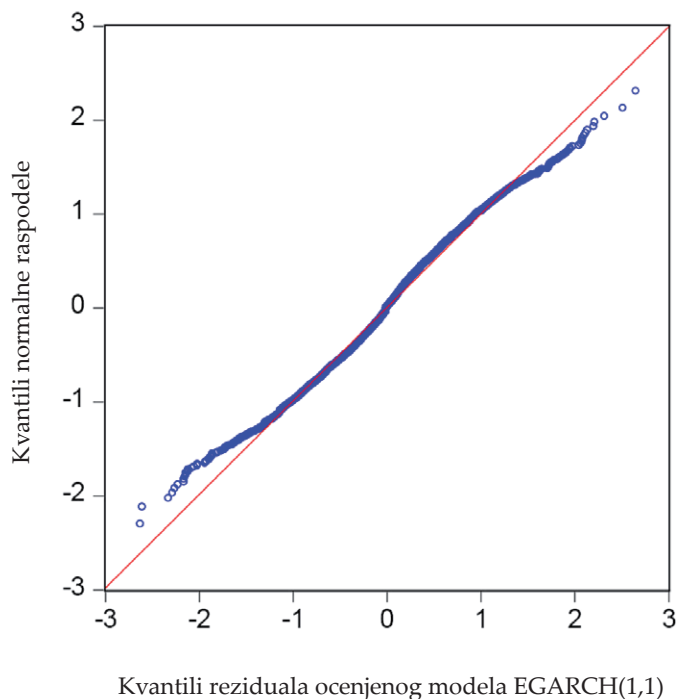
According to Table 6, it can be seen that the standardized residuals are not correlated (Ljung-Box test statistics - Q). This conclusion is indicated by the high p-value with the estimated Q statistic (at all lags $p > 0.05$), which means that one cannot reject the null hypothesis which states: There is no autocorrelation up to the order of k. Since the null hypothesis cannot be rejected, we conclude that there is no residual autocorrelation in the residuals of the estimated model. The correlogram of squared standardized residuals (Q2 statistics) also shows that the squared residuals are not autocorrelated, suggesting that the EGARCH model covers the dynamics of conditional variance in the satisfactory manner. Autocorrelation coefficients (AC) and partial autocorrelation (PAC) in the correlogram of standardized squared residuals are near zero at all lags, hence it can be concluded that there is no conditional autoregression heteroskedasticity in residuals at 0.05 level of significance.

The existence of ARCH effects can be checked by the ARCH statistics (Engle's LM test). This statistic is asymptotically distributed as a chi-square distribution if it cannot reject the null hypothesis that there is no heteroscedasticity in the residuals. The ARCH statistic was calculated by multiplying the coefficient of determination

for the model, in which the squared data are assessed in relation to the corresponding number of its previous value, by the volume of the sample. The calculated p-values in Table 6 related to the ARCH statistics are greater than 0.05 at all lags, so that the null hypothesis of the absence of ARCH effects in the residuals cannot be rejected. The sum of the estimated parameters α and β is greater than one, indicating the long-lasting impact of shocks to volatility.

Since the calculated JB statistics in Table 6 is greater than the critical value, and thereby the p-value to the fourth decimal is zero, the null hypothesis of normally distributed residuals is rejected at 0.05 level of significance (it should be noted that the quality of the model is not called into question due to deviations of the empirical distribution from the normal distribution, because it is not necessary that the random errors are normal distributed). Moreover, the QQ chart, which features a set of pairs of quantiles of the distribution of the series itself and the quantiles of the normal distribution, shows that there is a disagreement between the empirical and normal distribution on the ends (tails) of the series. As we can see, Figure 5 shows both positive and negative shocks, which cause a deviation of the empirical distribution of residuals from the normal distribution. We noted earlier that the tails of the empirical distribution of financial instruments are often heavier than the tails of the normal distribution (the empirical series of standardized residuals is normally distributed when its third moment is equal to zero, and fourth moment is equal to the value of 3). Although the corresponding quantile couples at the ends (higher values of shocks) do not coincide with the quantiles of the normal distribution, the empirical distribution of residuals in a large part of the series can be approximated by the normal distribution. Given the low value of the coefficient of skewness (-0.05), it can be said that the empirical distribution is symmetrical. However, due to the higher value of the kurtosis, which equals three, there is a deviation from normality.

Grafikon 5. QQ dijagram empirijske serije u odnosu na normalnu raspodelu



Izvor: Autor

Da bi se izabrala specifikacija koja pokazuje najbolja statistička svojstva, za sve modele koji su ocenjeni u ovom radu su izračunati AIC, SC i HQC (GARCH) informacijski kriterijumi. Smatra

se da najbolju specifikaciju ima model sa najmanjim vrednostima informacijskih kriterijuma. Na osnovu izračunatih vrednosti ovih kriterijuma u tabeli 7. može se zapaziti da su one najmanje kod EGARCH(1,1) modela. Takođe se uočava da ocenjene GARCH specifikacije imaju manju vrednost informacijskih kriterijuma od ARCH specifikacije. Dakle, potvrđuje se očekivanje da su GARCH modeli ekonomičniji od ARCH specifikacije. Osim GARCH(1,1) i EGARCH(1,1) modela, u ovom radu je ocenjeno još nekoliko GARCH modela. To su: GARCH(1,1)-M, TARCH(1,1), APARCH, Taylor-Schwert PARCH i CGARCH model. Ocene ovih modela nisu prikazane u radu, jer su oni bili manje uspešni u modeliranju nestabilne uslovne

varijanse. Međutim, vrednosti informacijskih kriterijuma iz ocene ovih modela date su u tabeli 7.

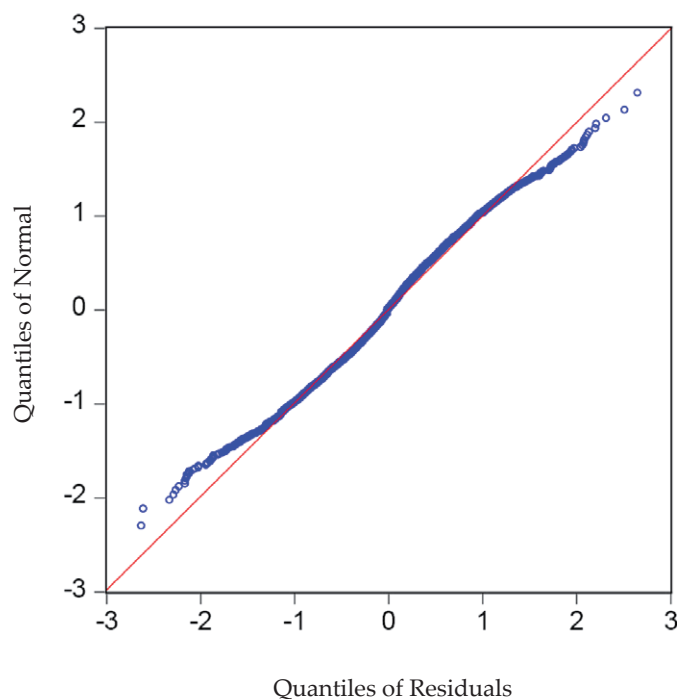
Tabela 7. Izbor modela na osnovu AIC, SC i HQC informacijskih kriterijuma

| Model | AIC | SC | H-QC (Hannan-Quinn) |
|----------------------------------|----------|----------|---------------------|
| 1. ARCH(5) sa veštačkim prom. | 1,860710 | 1,882958 | 1,868569 |
| 2. GARCH(1,1) | 1,797591 | 1,815389 | 1,803877 |
| 3. GARCH(1,1)-M | 1,797923 | 1,817204 | 1,804733 |
| 4. TARCH (1,1) model | 1,798130 | 1,817411 | 1,804940 |
| 5. APARCH model | 1,796136 | 1,816900 | 1,803470 |
| 6. Taylor-Schwert PARCH model | 1,796346 | 1,814144 | 1,802632 |
| 7. EGARCH(1,1) Normalna distrib. | 1,795304 | 1,814585 | 1,802114 |
| 8. CGARCH - bez člana asimetrije | 1,796358 | 1,817122 | 1,803692 |

Napomena: U tabeli su data tri informacijska kriterijuma: AIC ((Akaikeov (Akaike)), SC (Švarcov (Schwarz)) i H-QC (Hana-Kvinov (Hannan-Quinn)). U praksi se najčešće koristi SC kriterijum.

Izvor: Obrada autora pomoću softverskog paketa EViews.

Figure 5. QQ diagram of the empirical series against that of the normal distribution



Source: Author

To choose a specification that indicates the best statistical properties, AIC, SC and HQC (GARCH) information criteria were calculated for all models that were estimated in this paper. It is believed that the best specification is owned

by the model with the lowest values of information criteria. Based on the calculated values of these criteria, in Table 7 it can be observed that they are the lowest in the EGARCH (1,1) model. It can also be noted that the estimated GARCH specifications are less than the value of the information criteria of the ARCH specification. So, this confirms the expectation that the GARCH models are more economical than the ARCH specifications. In addition to the GARCH (1,1) and EGARCH (1,1) models, the author estimated several others GARCH models. These are: GARCH (1,1)-M, Tarch (1,1) APARCH, Taylor-Schwert PARCH and CGARCH model. However, the estimates of these models are not given in the paper, because they were less successful in modeling the unstable conditional variance. Nevertheless, the values of the information criteria of the estimated models are given in Table 7.

Table 7. The model selection based on AIC, SC and HQC information criteria

| Model | AIC | SC | H-QC (Hannan-Quinn) |
|---|----------|----------|---------------------|
| 1. ARCH(5) with dummy var. | 1,860710 | 1,882958 | 1,868569 |
| 2. GARCH(1,1) | 1,797591 | 1,815389 | 1,803877 |
| 3. GARCH(1,1)-M | 1,797923 | 1,817204 | 1,804733 |
| 4. TARCH (1,1) Model | 1,798130 | 1,817411 | 1,804940 |
| 5. APARCH Model | 1,796136 | 1,816900 | 1,803470 |
| 6. Taylor-Schwert PARCH Model | 1,796346 | 1,814144 | 1,802632 |
| 7. EGARCH(1,1) Normal distrib. | 1,795304 | 1,814585 | 1,802114 |
| 8. CGARCH - without the member of asymmetry | 1,796358 | 1,817122 | 1,803692 |

Note: Three information criteria: Akaike, Schwarz and H-QC (Hannan-Quinn) are given in the table. In practice, SC is the most frequently used criterion.

Source: Author by EViews software package.

Među ocenjenim GARCH modelima, eksponencijalni GARCH model pokazuje najmanje vrednosti AIC i HQC informacionih kriterijuma, dok je najmanja vrednost SC kriterijuma ostvarena kod Tejlor-Švertovog (Taylor-Schwert) PARCH modela. Takođe, ocenjena vrednost SC kriterijuma za EGARCH(1,1) model neznatno zaostaje za ocenjenom vrednošću ovog kriterijuma za Tejlor-Švertov PARCH model. S obzirom da su dva kriterijuma dala prednost EGARCH(1,1) modelu, a da kod SC kriterijuma EGARCH(1,1) neznatno zaostaje za superiornom specifikacijom Tejlor-Švertovog PARCH modela, zaključeno je da EGARCH(1,1) model predstavlja adekvatnu specifikaciju.

Zaključak

Učesnici na međunarodnom finansijskom tržištu i subjekti međunarodne trgovine izloženi su riziku deviznog kursa u savremenim uslovima globalizacije i liberalizacije svetskog tržišta. Što je privreda otvorenija, jači je uticaj međunarodnog okruženja na promene deviznog kursa. S obzirom na brojnost faktora koji utiču na promene deviznog kursa, prognoze njegove potencijalne volatilnosti su važne za projektovanje makroekonomskih varijabli i komponovanje mera ekonomske politike. Rizicima volatilnosti deviznog kursa posebno su izloženi investitori. Stoga je razumljiva njihova zainteresovanost za procene budućih oscilacija deviznog kursa, kako bi preduzeli odgovarajuće mere zaštite od ovih rizika. Da bi se kvantitativno modelirale oscilacije deviznog kursa, u finansijskoj analizi su razvijeni metodi koji mogu da odgovore ovom zadatku. Među njima važnu ulogu imaju ARCH i GARCH modeli. U ovom radu modelirana je vremenska serija dnevne stope prinosa deviznog kursa evra prema dolaru pomoću ARCH i nekoliko varijanti GARCH modela. U modeliranje ove vremenske serije uključeno je više veštačkih varijabli pomoću kojih su otklonjene ekstremne oscilacije u seriji. Ocenjeno je nekoliko GARCH modela sa različitim brojem parametara. Svi ocenjeni modeli su imali zadovoljavajuća

statistička svojstva. Zajedničko za ocene GARCH modela je činjenica da su dobijeni koeficijenti kvadriranih reziduala na donji i koeficijenti uz parametar uslovne varijanse u jednačini uslovne varijanse naglašeno statistički signifikantni. Zbir vrednosti ocene ova dva koeficijenta je bio blizu jedinice, što je tipično za ocenjene GARCH modele na podacima prinosa finansijske aktive. To je bio signal da će šokovi u jednačini uslovne varijanse dugo trajati. Na osnovu dobijenih vrednosti zbira ova dva koeficijenta kod razmatranih GARCH modela, koje su bile oko jedan, ocenjeno je da veće promene pozitivnog ili negativnog prinosa (šokovi) dovode do većih prognoziranih vrednosti varijanse u produženom periodu. Da bi se izabrao model koji se najbolje prilagođava ispitivanoj vremenskoj seriji, ocenjene su vrednosti standardnih informacionih kriterijuma. Polazeći od ovih ocena, nametnulo se zapažanje da je najbolje rezultate u modeliranju stope prinosa evra pokazao asimetrični EGARCH(1,1) model. Koeficijent asimetrije u jednačini volatilnosti kod ovog modela je negativan, i nije statistički signifikantan. Negativna vrednost ovog koeficijenta sugerise ocenu da se može očekivati da će negativni šokovi više uticati na buduću uslovnu varijansu nego pozitivne inovacije. Asimetrični EGARCH(1,1) model takođe je pružio dokaze o postojanju leveridž efekta, pokazujući da negativne informacije (šokovi) uslovljavaju veću volatilnost u narednom periodu nego pozitivni šokovi. Rezultati primenjenih modela pokazuju da je uticaj šokova na volatilnost prinosa deviznog kursa evra prema dolaru veoma jak i dugotrajan. Sem toga, volatilnost prinosa deviznog kursa pokazuje asimetrične efekte, pri čemu je uticaj negativnih šokova jači od uticaja pozitivnih šokova. Dobijeni rezultati su veoma značajni za investitore jer sugerisu povećanu opreznost u poslovanju sa evrom, posebno u slučaju makro šokova. Dalja istraživanja bi trebalo usmeriti na ispitivanje asimetričnog ponašanja drugih deviznih kurseva u svetskoj privredi.

Among the estimated GARCH models, the exponential GARCH model shows the minimum value of AIC and HQC information criteria, while the lowest values of the SC criteria were recorded by the Taylor-Schwert PARCH model. Also, the estimated value of the SC criterion for EGARCH (1,1) model is slightly lower than the estimated value of this criterion for Taylor-Schwert PARCH model. Given that the two criteria give priority to EGARCH (1,1) model, and that in the SC criteria EGARCH (1,1) is slightly inferior to the superior specification of the Taylor-Schwert PARCH model, it was concluded that the EGARCH (1,1) model represents an adequate specification.

Conclusion

The participants in the international financial market and trade-engaged international business are exposed to the foreign exchange risk in the contemporary phase of the world market's globalization and liberalization. The more the economy is open, the stronger the influence of the international environment on the exchange rate changes. Given the large number of factors affecting the exchange rate changes, the forecasts of its potential volatility are important for the macroeconomic projections and the structure of economic policy measures. Investors are particularly exposed to the risk of the exchange rate volatility. Therefore, their interest in the estimates of future exchange rate fluctuations is understandable, since they can help them take the appropriate measures to protect themselves against these risks. To quantitatively model the exchange rate fluctuations, the relevant methods that can respond to this task have been developed in the financial literature. The ARCH and GARCH models play an important role among them. The time series of daily returns of the euro exchange rate against the dollar in this paper is modeled by several ARCH and GARCH models. Several dummy variables were included in the modeling of this series, whereby the extreme fluctuations in the series were removed. Several GARCH models were estimated with a different number of parameters. All estimated models

had the satisfactory statistical properties. A common feature of the estimated GARCH models is that the obtained coefficients of the lag squared residuals and coefficients of the conditional variance in the equation of conditional variance are statistically significant. The sum of these two estimates was close to a unit, which is typical for the financial assets returns time series in the estimated GARCH models. It was a signal that the innovations in the conditional variance equation will be long-lasting. Based on the obtained value of the sum of these two coefficients, which were about one, it is estimated that the major changes in the positive or negative returns (shocks) are leading to the higher forecasted value of variance in the extended period. In order to choose the best fitting model, the values of the standard information criterion were estimated. According to these assessments, it was clear that the asymmetrical EGARCH (1,1) model showed the best results in the modeling of the euro-dollar exchange rate returns. The coefficient of asymmetry in the conditional variance equation of this model is negative and is not statistically significant. A negative value of this ratio suggests that negative shocks will have a bigger impact on the future conditional variance than the positive innovation. The asymmetric EGARCH (1,1) model has also provided evidence of the leverage effect, showing that the negative information (shocks) generate the greater volatility in the coming period than the positive shocks. The results obtained from these models indicate that the impact of shocks on the volatility of the euro exchange rate against the dollar is very strong and long-lasting. In addition, the volatility of the exchange rate shows the asymmetric effects, whereby the impact of negative shocks is stronger than the impact of positive shocks. The obtained results are very important for investors because they suggest that the degree of cautiousness in dealing with the euro should be increased, particularly in the case of macroeconomic shocks. Further research should examine the asymmetric behavior of other foreign exchange rates in the global economy.

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