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BOOTSTRAP MODEL ISTORIJSKE SIMULACIJE VREDNOSTI PRI RIZIKU

Prevod obezbedili autori

Rezime

U radu autori predstavljaju novi model VaR za procenu tržišnog rizika u bankama i drugim finansijskim institucijama. Model je označen kao BootstrapHS500, budući da se zasniva na teorijskim osnovama istorijske simulacije VaR i primeni bootstrap metode. Glavni cilj rada jeste da se dobije odgovor na pitanje da li inkorporiranje bootstrap metoda u standardni model istorijske simulacije vrednosti pri riziku dovodi do unapređenja aplikativnosti istorijske simulacije u kontekstu zadovoljenja pravila validnosti internih modela uz procenu tržišnog rizika Bazel II standarda. Kako bi se odgovorilo na ovo pitanje u radu je izvršeno testiranje aplikativnosti i komparacija performansi HS500 i BootstrapHS500 modela na tržištu kapitala Srbije, Hrvatske, Grčke, Španije, Nemačke, Slovačke, Češke, Rumunije i Mađarske. Metodologija istraživanja podrazumeva primenu odgovarajuće kvantitativne analize i testa bezuslovnog i uslovnog pokrića. Rezultati istraživanja pokazuju da BootstrapHS500 model, na testiranim tržištima, postiže bolje performanse iz vizure oba testa, te da se može zaključiti da inkorporiranje bootstrap metode u standardni model istorijske simulacije doprinosi unapređenju njegove aplikativnosti.

Ključne reči: Vrednost pri riziku, istorijska simulacija, bootstrap metod, tržišni rizik

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BOOTSTRAP HISTORICAL SIMULATION

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Summary

In this paper the authors present a new VaR model for the estimation of market risk in banks and other financial institutions. The model is labeled BootstrapHS500, since it is theoretically based on historical simulation and implementation of the bootstrap method. The aim of the paper is to provide answers to the question whether incorporating the bootstrap method in the standard model of historical simulation contributes to the improvement of the historical simulation's applicability in terms of meeting the backtesting rules of the Basel II standards. In order to obtain the answer to this question, in this paper we conducted and dealt with the testing of applicability and comparison of performances of the HS500 and the BootstrapHS500 models at the capital markets of Serbia, Croatia, Greece, Spain, Germany, Slovakia, the Czech Republic, Romania and Hungary. The research methodology involves the use of the appropriate quantitative analysis and tests of unconditional and conditional coverage. The results of the research show that the BootstrapHS500 model achieved a better performance than the standard model of historical simulation, from the perspective of both backtesting tests. It can be concluded that incorporating the bootstrap method in the standard model of historical simulation contributes to the improvement of the historical simulation' applicability.

Keywords: Value at Risk, historical simulation, bootstrap method, market risk

JEL: C22, C52, C53, G24

Uvod

Iako validnost procene vrednosti pri riziku (Value at Risk - VaR) kod svakog modela VaR zavisi od adekvatnog izbora uzorka, kod istorijske simulacije on predstavlja presudnu determinantu validnosti procene tržišnog rizika. Dva su osnovna razloga za to: 1) pretpostavka o konstantnoj distribuciji na kojoj model počiva i 2) odsustvo evaluacije faktora rizika. Model se zasniva na pretpostavci da kretanje osnovnih faktora rizika u prethodnim periodima sadrži sve potrebne informacije za procenu VaR, što je ekvivalentno pretpostavci o konstantnoj distribuciji. Pored toga, u građenju distribucije prinosa portfolija ne polazi se od teorijske distribucije, čime je izbegnut problem parametrizacije, ali i odsustvo evaluacije faktora rizika. Implikacije ovoga su jasne: procena VaR će biti potcenjena ako se kooptira suviše miran period volatilnosti, odnosno precenjena ako se kooptira period izuzetne volatilnosti. Otuda, efikasna primena modela podrazumeva, izbor perioda uzorkovanja koji će adekvatno reprezentovati distribuciju budućih prinosa portfolija, što je ekvivalentno pretpostavci o konstantnoj distribuciji, sa jedne strane, ali i dovoljan broj opservacija, sa druge, da bi se dobila statistički značajna procena kvantila distribucije (VaR). Međutim, upravo ovo predstavlja opasnost od kršenja pretpostavke o konstantnoj distribuciji. Pored toga, treba imati u vidu i činjenicu da u uslovima vremenski promenljive volatilnosti postoji limit za koliko istorijski podaci mogu biti relevantni za predviđanje volatilnosti Finger (2006). U takvoj situaciji uključivanje većeg broja opservacija neće doprineti unapređenju predviđanja (Radivojevic et al (2015)). Prethodno rečeno implicira da efikasna primena modela podrazumeva optimalan balans između dugog perioda postaranja, koji potencijalno krši pretpostavku o konstantnoj distribuciji i kratkog, koji dovodi do minimiziranja statističke značajnosti predviđanja modela

Međutim, izborom adekvatnog perioda uzorkovanja, problem efikasne primene modela se ne rešava u potpunosti jer jedan od nedostataka istorijske simulacije jeste nemogućnost pouzdane procene VaR za ekstremno visoke nivoe poverenja za holding periode duže od jednog dana[1], usled naglog smanjena broja opservacija. Primera radi uzorak od 500 dana dnevnih podataka brzo se smanji na uzorak sa 50 opservacija desetodnevih podataka o prinosu portfolija. U ovom slučaju skoro je nemoguće proceniti VaR za ekstremno visok nivo poverenja. Sa ovakvim uzorkom realna procena VaR moguća je tek za nivo poverenja od 98%, budući da se u tom slučaju procena VaR poklapa sa najvećim istorijskim gubitkom iz uzorka. Stvarna verovatnoća, pri normalnim tržišnim uslovima, da se ponovo zabeleži ekstremni gubitak svakako je manja od (1-nivo poverenja).

Nezavisno od ovoga, istraživanja Finger-a (2006), Rossignolo et al (2012, 2013), Zikovic-a (2007, 2010), Radivojevića i saradnika (2010b, 2015), pokazuju da postoji značajan obim greške u proceni VaR kada je model podešen prema zahtevima Bazel II standarda. Rezultati ovih istraživanja dovode u pitanje da li je uzorak od 250 podataka o dnevnim prinosima, koji je propisan Bazel II standardom, dovoljan da se proceni rep distribucije za nivo poverenja od 99%. Otuda, cilj ovog rada jeste da se ispita da li inkorporiranje bootstrap metoda u standardni model istorijske simulacije dovodi do unapređenja njegove aplikativnosti, u kontekstu zadovoljenja pravila Bazelskog komiteta vezana za validnosti internih modela VaR. Iznošenjem ovih ideja, u stvari, autori promovišu novi model VaR.

Teorijske osnove bootstrap istorijske simulacije

Jedan od načina na koji je potencijalno moguće prevazići u uvodnom delu iznete nedostatke standardnog modela istorijske simulacije nalazi se u primeni bootstrap metode. Polazeći od stava Babu-a i Singh-a (1983), da se primenom bootstrap metode uspešno može aproksimirati populacija reuzorkovanjem

1 Istraživanje Hendricks-a (1996) najbolje otkriva odnos između dužine holding perioda i validnosti procena VaR za različite modele VaR.

Introduction

Although the validity of the value at risk (Value at Risk - VaR) estimation with each VaR model depends on the adequate sampling, in case of the historical simulationit is the crucial determinant of the validity of the market risk estimation. There are two main reasons for this: 1) the assumption of a constant distribution on which the model is based and 2) the absence of estimation of risk factors. The model is based on the assumption that the movement of the underlying risk factors in the previous periods contains all the information necessary for assessing the VaR, which is equivalent to the assumption of a constant distribution. In addition, creating the distribution return of the portfolio does not begin with the theoretical distribution, which circumvents the problem of parameterization, but also the lack of estimation of risk factors. The implication is clear: VaR estimates will be underestimated if a too quiet period of volatility is coopted, or overestimated if a period of exceptional volatility is coopted. Hence, the effective application of the model implies the choice of the sampling period that will adequately represent the distribution of future returns of the portfolio, which is equivalent to the assumption of a constant distribution on the one hand, and a sufficient number of observations, on the other, to obtain a statistically significant estimation of the quantile of distribution (VaR). However, this is precisely the danger of a breach of the presumption of constant distribution. In addition, we should bear in mind the fact that in terms of the time-varying volatility, there is a limit to how much historical data may be relevant for predicting volatility (Finger, 2006). In such a situation a larger number of observations will contribute to improving predictions (Radivojevic et al (2015)). The foregoing implies that the effective application of the model involves the optimal balance between the long period of aging, which potentially violates the assumption of the constant distribution, and the short one, which leads to the minimization of statistical significance of the model's prediction.

However, the choice of an adequate sampling period does not completely solve the problem of effective implementation of the model because one of the shortcomings of historical simulation VaR is the inability to make reliable estimates for extremely high levels of trust for holding periods longer than one day[1], due to a sudden decrease of the number of observations. For example, the sample of 500 days of daily data quickly reduces to the sample with 50 observations of the ten-day data on the portfolio's return. In this case it is almost impossible to estimate VaR for the extremely high confidence level. With this pattern, a realistic assessment of VaR is only possible for a confidence level of 98%, since in this case the VaR estimate coincides with the biggest historical loss from the sample. The actual probability under normal market conditions, to re-record the extreme loss is certainly less than (1-level of confidence).

Apart from that, the research done by Finger (2006), Rossignol et al (2012, 2013), Zikovic (2007, 2010), Radivojevic and the associates (2010b, 2015), show that there is a significant volume of errors in estimating VaR when the model is set according to the requirements of Basel II. The results of these studies call into question whether a sample of 250 data on the daily returns, prescribed by the Basel II standards, is sufficient to evaluate the distribution tail for the confidence level of 99%. Hence, the aim of this study is to investigate whether the incorporation of the bootstrap method in the standard historical simulation model leads to the improvement of its applicability, in the context of meeting the Basel Committee rules related to the validity of internal VaR model. By presenting these ideas, the authors, in fact, promote a new VaR model.

Theoretical background of the bootstrap historical simulation

One of the ways that may potentially help in overcoming the shortcomings of the standard

1 Hendricks's research (1996) reveals the relationship between the length of the holding period and the validity of the VaR estimates for different VaR models.

velikog broja (koji teži ka beskonačnosti) uzoraka populacije, u radu se predlaže novi model istorijske simulacije. Model predstavlja direktnu ekstenziju standardnog modela istorijske simulacija i označen je kao *BootstrapHS* model. Dizajniran je tako da reši problem pouzdane procene VaR za ekstremno visok nivo poverenja za veličinu uzorka definisanu Bazel II standardom, budući da brojna istraživanja ukazuju na ovaj nedostatak istorijske simulacije. Ideja je da se u standardni model istorijske simulacije inkorporora bootrap pristup, kako bi se efikasno aproksimirala funkcija gustine verovatnoće (*probability density function - PDF*) populacije osnovnih faktora tržišnog rizika, koja po pravilu odstupa od normalne distribucije.

Primena bootstrap pristupa je jednostavna i zavisi od toga da li je poznat oblik funkcije raspodele prinosa portfolija sa nekim nepoznatim parametrom koji figuriše u njoj $\theta = \theta(F)$ ili nije. U zavisnosti od ovoga moguća je primena parametarske ili neparametarske bootstrap metode.

Predloženi model jednostavan je za razumevanje i lak za implementaciju. Implementacija modela vrši se u tri koraka. Prvi korak podrazumeva generisanje istorijskih prinosa veliki broj puta (M) primenom bootstrap metode. Model zadržava istu strukturu serije istorijskih prinosa (r_t), pri čemu se ona tretira kao populaciju, i nasumično bira istorijske podatke koji se reuzorkuju (M) puta. Drugim rečima, istorijski prinosi se simuliraju kao vektor slučajnih r_t ishoda na osnovu seta podataka θ. Na taj način dobija se (M) novih simuliranih uzoraka sa N brojem opservacija.

Nakon ovog koraka procena VaR za svaki simulirani (M) uzorak vrši se na identičan način kao i kod standardnog modela istorijske simulacije. Procena VaR svodi se na jednostavno utvrđivanje vrednosti (N + 1)cl člana uređenog niza podataka o prinosima portfolija, pri čemu (*cl*) predstavlja nivo poverenja za koji se vrši procena VaR. Ukoliko (N + 1)cl nije celobrojna vrednost, primenjuje se pravilo interpolacije između dve susedne opservacije.

Prethodno rečeno, matematički može se iskazati na sledeći način:

$$VaR^{cl}_{N=1|N} \equiv r_w((N+1)cl)$$

pri čemu je r_w ((N + 1)cl) preuzet iz uređenog niza prinosa {r_w (1), r_w (2) …r_w (N)}.

Poslednji korak podrazema utvrđivanje VaR portfolija kao prosečne vrednosti procena VaR dobijenih za (M) simuliranih serija prinosa:

$$Va\overline{R}_{cl} = \frac{1}{M}\sum_{i=1}^{M} VaR_i$$

Ovako dobijen model otklanja, ili preciznije reći, ublažava neka ograničenja standardnog modela istorijske simulacije, pre sve vezana za nedovoljan broj opservacija i procenu VaR za ekstremne nivoe poverenja za holding periode duže od jednog dana. Kao i u slučaju standardnog modela i kod BootstrapHS modela svakom opažaju dodeljuje se isti ponder. To znači da se pretpostavlja da svaki opažaj sa istim intenzitetom utiče na procenu VaR. Problem koji nastaje primenom ovakvog načina ponderisanja, kod istorijske simuacije, jeste pojava efekta "duha". Reč je o pojavi da ekstremni gubici koji su se dogodili u daljoj prošlosti zbog dugog vremenskog perioda korišćenog u istorijskoj simulaciji kontinuirano utiču na procenu VaR, a zatim naglo nestaju kako ispadaju iz izabranog perioda posmatranja (Radivojević et al (2010a)). Međutim, kod predloženog modela reuzorkovanjem veliki broj puta ovaj problem, u najmanju ruku, se značajno ublažava. Upravo zbog postanja ekstremnih prinosa tokom perioda uzorkovanja, često se dešava da standardni model ispunjava kriterijume bezuslovnog pokrića, što pruža lažan osećaj sigurnosti (detaljnije o ovom fenomenu videti u Radivojević et al 2015).

Međutim, ovako specificiran model opterećen je svim nedostacima i organičenjima vezanim za primenu bootstrap pristup (detaljnije videti Hall, 1994).

Podaci i metodologija istraživanja

Za testiranje validnosti modela korišćeni su dnevni logaritamski prinosi berzanskih indeksa sa tržišta kapitala Srbije, Hrvatske, Grčke, Španije, Nemačke, Slovačke, Češke, Rumunije i Mađarske. Testirani indeksi su BELEX15 (Srbija), CROBEX10 (Hrvatska), ATHES20 (Grčka), IBEX35 (Španija), DAX30 (Nemačka), SAX (Slovačka), PX (Češka), BET (Rumunija) model of historical simulation presented in the introduction is the application of bootstrap methods. Starting from the premise of Babu and Singh (1983), that the population can be successfully approximated with the application of a bootstrap method by resampling a large number (which strives to infinity) of population samples, the paper proposes a new model of historical simulation. The model is a direct extension of the standard model of historical simulation and is marked as the BootstrapHS model. It is designed to solve the problem of reliable VaR estimates for extremely high level of trust for the sample size defined by Basel II standards, since numerous studies point to the shortcoming of historical simulation. The idea is to incorporate the bootstrap approach into the standard model of historical simulation, in order to effectively approximate the probability density function -PDF of the population of the basic market risk factors, which, as a rule deviates from the normal distribution.

The application of the bootstrap approach is simple and depends on whether a known form of the distribution function of portfolio return contains some unknown parameter in it $\theta = \theta(F)$ or not. Depending on this it is possible to apply a parametric or non-parametric bootstrap method.

The proposed model is easy to understand and easy to implement. The implementation of the model is conducted in three steps. The first step involves generating historical yield many times (M) using the bootstrap method. The model retains the same structure of the series of r historical returns (r_t), wherein it is treated as population, and randomly choses historical data which are resampled (M) a number of times. In other words, historical returns are simulated as the vector of random r_t outcome based on the set of data θ. This is how we get (M) new simulated samples with the N number of observations.

After this step VaR estimation for each simulated (M) sample is done in an identical way as in the standard model of historical simulation. VaR estimation comes down to a simple determination of values $(N+1)$cl member of the ordered series of data on portfolio returns, where (*cl*) represents the level of confidence for which VaR estimation is done. If $(N + 1)$cl is not an integer, the rule of interpolation is applied between two neighboring observations. The previously said can be mathematically expressed in the following way:

$$VaR^{cl}_{N=1|N} \equiv r_w((N+1)cl)$$

in which case r_w $((N + 1)\text{cl})$ taken from the ordered series of returns$\{r_w(1), r_w(2)\ldots r_w(N)\}$.

The last step is the determination of the portfolio VaR as the average value of VaR estimate obtained for (M) simulated series of returns:

$$\overline{VaR}_{cl} = \frac{1}{M}\sum_{i=1}^{M} VaR_i$$

Thus obtained model eliminates, or more precisely, relieves some of the limitations of the standard model of historical simulation, all related to the insufficient number of observations and the VaR estimation for extreme levels of confidence for holding periods longer than one day. As in the case of the standard model, the Bootstrap HS model also has the same weight assigned to each observation. This means that it is assumed that any observation with the same intensity affects the VaR estimation. The problem that arises by using this method of weighting, with the historical simulation, is the appearance of the 'ghost effect'. It is a phenomenon in which extreme losses that occurred in the distant past continuously affect the estimation of VaR because of the long time period of use in the historical simulation, and then suddenly disappear when they fall out of the chosen observation period (Radivojevic et al (2010a)). However, in the proposed model this problem can, at the very least, be solved by doing the resampling a great number of times. Because of the extreme returns during the sampling period, it often happens that the standard model meets the criteria of unconditional coverage, which provides a false sense of security (more on this phenomenon in Radivojevic et al, 2015).

However, such a specified model is loaded with all the deficiencies and limitations related to the use of bootstrap approach (for more details see Hall, 1994).

i BUX (Mađarska). Podaci su prikupljeni sa zvaničnih sajtova za period od 24.01.2013. do 19.01.2016. godine. Logaritamski prinosi generisani su na sledeći način:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$$

pri čemu su:

$r_{i,t}$ - logaritamski prinos za dan (t)

$P_{i,t}$ - vrednost posmatranog indeksa na zatvaranju za tekući dan

$P_{i,t-1}$ - vrednost posmatranog indeksa na zatvaranju prethodnog dana

Dnevne procene VaR načinjene su za period od 19.01.2015. do 19.01.2016. godine, što je u skladu sa pravilima Bazel II standarda, za nivoe poverenja od 99 i 99,5%. Da bi se obezbedio isti uzorak za testiranje validnosti modela za svako tržište ovaj period je uzet kao period za testiranje validnosti modela. Ostatak opservacija upotrebljen je za dobijanje inicijalnih procena VaR.

Kao reprezent standardnog modela istorijske simulacije, korišćen je HS500 model. Procene VaR primenom ovog modela utvrđene su kao kvantil empirijske distribucije prinosa. Tako je procena VaR za dan 19.01.2015. godine (prvi dan za koji se vršila procena rizika) dobijena primenom prvih 500 opservacija. Za naredni dan, 20.01.2015. godine, procena je dobijena na taj način što je iz uzorka isključena prva, a u uzorak je uključena 501. opservacija. Na taj način dobijena je procena tržišnog rizika za 20.01.2015. godinu. Postupak je tako ponovljen 244 puta. Na taj način dobijene su 244 procene tržišnog rizika. Kao reprezent BootstrapHS modela u radu korišćen je BootstrapHS500 model. Procene VaR primenom ovog modela utvrđene su kako je opisano u prethodnom delu rada.

Rezultati testiranja validnosti modela

U ovom delu rada predstavljeni su i analizirani rezultati testiranja validnosti modela. Modeli su procenjeni u pogledu tačnosti njihovih procena *VaR* tokom poslednjih 244 dana perioda posmatranja. Ovaj period označen je kao period testiranja validnosti modela. Svaki model testiran je na sledeći način: dnevne procene VaR, koje su dobijene za nivoe poverenja od 99,5% i 99%, poređene su sa stvarnim kretanjima prinosa u periodu od 19.01.2015. do 19.01.2016. godine. U slučaju kada je stvarni gubitak bio veći od procene VaR načinjene za taj dan, evidentirano je da se desilo prekoračenje. Zatim je utvrđen ukupan broj/procenat prekoračenja tokom perioda testiranja validnosti.

Prema Jorion-u (2007) model je validan ukoliko je broj prekoračenja jednak (1-cl). U ovom slučaju, to znači da ukupan broj prekoračenja, tj. dana kada je stvarni gubitak bio veći od procene VaR načinjene za taj dan, ne sme da bude veći od 2 prekoračenja za nivo poverenja od 99%, odnosno od 1 za nivo poverenja od 99,5%. Drugim rečima, procenat prekoračenja VaR ne sme da pređe 1%, odnsno 0,5% od ukupnog broja procena VaR.

Broj, odnosno procenat prekoračenja VaR tokom perioda testiranja validnosti modela od 19.01.2015. do 19.01.2016. godine, posebno za svaki model i za svako tržište, prikazani su u tabeli 1.

Tabela 1. Broj/procenat prekoračenja

	HS500 VaR 99%		HS500 VaR 99, 5%		BootstrapHS500 VaR 99%		BootstrapHS500 VaR 99, 5%	
Indeksi	Br. prekoračenja	%	Br. prekoračenja	%	Br. prekoračenja	%	Br. prekoračenja	%
BELEX15	5	2,05	4	1,64	4	1,64	3	1,23
CROBEX10	3	1,23	1	0,41	2	0,82	0	-
ATHEX20	5	2,05	2	0,82	6	2,45	4	1,64
IBEX35	5	2,05	4	1,64	6	2,45	5	2,05
DAX30	9	3,69	7	2,87	7	2,86	6	2,45
SAX	1	0,41	1	0,41	1	0,41	0	-
PX	4	1,64	4	1,64	4	1,64	3	1,23
BET	4	1,64	3	1,23	4	1,64	3	1,23
BUX	3	1,23	1	0,41	3	1,23	1	0,41

Izvor: Autori

Kao što se može videti na osnovu rezultata prikazanih u tabeli 1, za nivo poverenja od 99%, HS500 model uspeo je da zadovolji Jorion-ov kriterijum samo na tržištu kapitala Slovačke.

Data and the methodology of analysis

The daily logarithmic returns of the stock indexes from the capital markets of Serbia, Croatia, Greece, Spain, Germany, Slovakia, the Czech Republic, Romania and Hungary were used for the performance analyses of the model. The tested stock indexes are the BELEX15 (Serbia), CROBEX10 (Croatia), ATHES20 (Greece), IBEX35 (Spain), DAX30 (Germany), SAX (Slovakia), PX (the Czech Republic), BET (Romania) and BUX (Hungary). The returns are collected from the official stock exchange websites of these countries for the period between January 24th 2013 and January 19th 2016. **The daily returns of selected stocks indexes are generated using the logarithmic approximation:**

$$r_{i,t} = log\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \qquad (1)$$

where

$r_{i,t}$ - log revenue on the day (t)

$P_{i,t}$ - the closing value of the observed index *i* on the day (t)

$P_{i,t-1}$ - the closing value of the observed index *i* on the day before (t-1).

The calculated VaR figures are for a one-day ahead horizon for the period between January 19th 2015 and January 19th 2016, which is in accordance with the Basel accord. To secure the same out-of-the-sample VaR backtesting period for all the tested stock indexes, the out-of-the-sample data sets were formed by taking out 244 of the latest observations for each stock indexes. The rest of the observations were used as resample observations needed for the VaR starting values.

As the representative of the standard model of historical simulation, the HS500 model was applied. By applying this model, the VaR estimates were obtained as a percentile of the empirical distribution of returns. For each analyzed stock index, VaR estimates were performed in the following way: first, we estimated one-day-ahead horizon VaR using the returns from the first 500 days. In that way we obtained VaR estimate for January 20th 2015 (estimate VaR for 501st day from the observed period). Then, for the next day, January 21th 2015, we used the returns for the past 500 days (covering the period from 2nd to 501st day). Actually, in that way we got the estimate VaR for 502nd day from the observed period. The process was repeated for all 244 days and obtained 244 VaR estimates for the selected period. As the representative of the BootstrapHS, the BootstrapHS500 model was applied. By applying this model, the VaR estimates were obtained as described in the preceding part of the paper.

The backtesting results

In this section the backtesting results for the tested models are presented, analyzed and discussed. The tested VaR models were evaluated in terms of their accuracy in estimating VaR over the last 244 days in the observed period. We called this period the backtesting period. Each model was tested as follows: first, the daily VaR estimates, obtained at a 99% and 99,5% confidence levels, were compared with the actual losses that occurred in the backtesting period (period between January 19th 2015 and January 19th 2016). In cases where the actual loss on a particular day exceeds VaR estimate for that day, we conclude that a VaR break occurred. Then the number/percentage of VaR breaks over the backtesting period of 244 days is counted. According to Jorion (2006), in a good model the percentage of VaR breaks should be equal to one minus the level of confidence. In this case, it means that the number of VaR breaks must not exceed 2 at a 99% confidence level (1% of VaR estimates total number), i.e. not more than 1 VaR breaks at a 99,5% confidence level (0,5% of VaR estimates total number).

The number/percentage of VaR breaks over the backtesting period (period between January 19th 2015 and January 19th 2016) separately for each of the selected stock indexes, is given in Table 1.

Na svim ostalim tržištima generisao je veći broj prekoračenja od očekivanog. Za nivo poverenja od 99,5%, model generisao je bolje procene VaR. U ovom slučaju model je zadovoljio Jorion-ov kriterijum u slučaju tri tržišna indeksa: CROBEX10, SAX i BUX. Zanimljivo je da je, za oba nivoa poverenja, model zadovoljio Jorion-ov kriterijum na tržištu kapitala Slovačke. Sa druge strane, iznenađujuće je da model nije zadovljio kriterijum na tržištu kapitala Nemačke, budući da je reč o razvijenom finansijskom tržištu, za razliku od ostalih, koja su tržišta u nastajanju i za koja je očekivano da će model da pokaže slabije performanse jer karakteristike tržišta nisu kompatibilne sa teorijskim osnovama na kojima počiva istorijska simulacija. BootstrapHS500 model, za nivo poverenja od 99%, zadovoljio je Jorion-ov kriterijum, takođe samo u slučaju indeksa SAX. Za nivo poverenja od 99,5%, model je zadovoljio Jorion-ov kriterijum u slučaju tri tržišna indeksa: CROBEX10, SAX i BUX. Međutim, samo na osnovu ovog kriterijuma i podataka prikazanih u tabeli 1 ne može se doneti validan zaključak da li bootstrap metoda doprinosi unapređenju aplikativnosti standardnog modela istorijske simulacije, budući da je BootstrapHS500 model na na nekim tržišta generisao manji, a na drugim veći broj prekoračenja u odnosu na HS500 model.

Da bi se formalno ispitalo da li se testirani modeli mogu smatrati validnim u skladu sa pravilima Bazel II standarda, odnosno da bi se odgovorilo na gore postavljeno pitanje, neophodna je primena nekog formalnog testa za ispitivanje validnosti modela VaR. Za tu svrhu u radu korišćeni su Kupiec-ov test bezuslovnog pokrića i Christofferson-ov test uslovnog pokrića. Validnost modela testirana je za nivo značajnosti testa od 95% jer za ovaj nivo značajnosti ova dva testa generišu jasne dokaze o koristi modela i implicira da se model odbaci samo ako za to postoje snažni dokazi (Samanta, G. P. et al. (2010)). Rezultati Kupiec-ovog testa bezuslovnog pokrića, za nivo poverenja od 95%, prikazani su u tabeli 2.

Tabela 2. Rezultati Kupiec-ovog testa bezuslovnog pokrića

	HS500 VaR 99%		HS500 VaR 99, 5%		BootstrapHS500 VaR 99%		BootstrapHS500 VaR 99, 5%	
Indeksi	kritična vrednost	p-vrednost	kritična vrednost	p-vrednost	kritična vrednost	p-vrednost	kritična vrednost	p-vrednost
BELEX15	2,082	0,149	3,949	0,047	0,844	0,358	1,837	0,175
CROBEX10	0,121	0,728	0,044	0,833	0,089	0,765	-	-
ATHEX20	2,082	0,149	0,413	0,520	3,700	0,054	3,949	0,047
IBEX35	2,082	0,149	3,949	0,047	3,700	0,054	6,574	0,010
DAX30	10,554	0,001	12,989	0,000	5,683	0,017	9,610	0,002
SAX	1,105	0,293	0,044	0,833	1,105	0,293	-	-
PX	0,844	0,358	3,949	0,047	0,844	0,358	1,837	0,175
BET	0,844	0,358	1,837	0,175	0,844	0,358	1,837	0,175
BUX	0,121	0,728	0,044	0,833	0,121	0,728	0,044	0,833

Izvor: Autori

Kao što se može videti u tabeli 2, za nivo poverenja od 99%, Kupiec-ov test bezuslovnog pokrića HS500 model nije zadovoljio samo na tržištu kapitala Nemačke, dok za nivo poverenjaod 99,5% nije prošao test čak na četiri tržiša i to: Srbije, Španije, Nemačke i Slovačke. Sa druge strane, BootstrapHS500 model zadovoljio je kriterijum bezuslovnog pokrića na svi tržištima za nivoa poverenja od 99%, dok za nivo poverenja nije zadovoljio kriterijum na tržištu kapitala Grčke, Španije i Nemačke. Na osnovu ovog kriterijuma može se izneti zaključak da inkorporiranje bootstrap metode u standardni model istorijske simulacije doprinosi unapređenju aplikativnosti istorijske simulacije, u kontekstu pravila validnosti Bazel II standarda.

Teorijski posmatrano, ekstremni gubici koji su se dogodili pre ili tokom perioda testiranja validnosti modela, mogu da utiču da procene VaR budu veoma visoke i na taj način se ostvaruje zadovoljavajuće bezuslovno pokriće, bez uzimanja u obzir stvarnog nivoa rizika. Iz tog razloga, na tržištima za koja su modeli prošli Kupiec-ev test bezuslovnog pokrića, neophodno je sprovesti i test uslovnog pokrića. Drugim rečima, da bi se potvrdio prethodno izneti zaključak, u radu je izvršeno testiranje validnosti modela i sa aspekta uslovnog pokrića, uz napomenu da zbog nesavršenosti

Table 1. The number/percentages of VaR breaks at 95% and 99% confidence levels

	HS500 VaR 99%		HS500 VaR 99, 5%		BootstrapHS500 VaR 99%		BootstrapHS500 VaR 99, 5%	
Stock index	No. of VaR breaks	%	No. of VaR breaks	%	No. of VaR breaks	%	No. of VaR breaks	%
BELEX15	5	2.05	4	1.64	4	1.64	3	1.23
CROBEX10	3	1.23	1	0.41	2	0.82	0	-
ATHEX20	5	2.05	2	0.82	6	2.45	4	1.64
IBEX35	5	2.05	4	1.64	6	2.45	5	2.05
DAX30	9	3.69	7	2.87	7	2.86	6	2.45
SAX	1	0.41	1	0.41	1	0.41	0	-
PX	4	1.64	4	1.64	4	1.64	3	1.23
BET	4	1.64	3	1.23	4	1.64	3	1.23
BUX	3	1.23	1	0.41	3	1.23	1	0.41

Source: Authors' calculations

As can be seen from the results shown in Table 1, at a confidence level of 99%, HS500 model satisfied the Jorion's criteria only in the case of the capital market of Slovakia. In cases of all other markets, the model generated higher percentages of VaR breaks than the expected number. At the confidence level of 99.5%, the model showed better performances. At this confidence level, the model satisfied the Jorion's criteria in case of three indexes: CROBEX10, SAX and BUX. It is interesting that, at both confidence levels, the model satisfied the Jorion's criteria on the Slovakian capital market. On the other hand, it is surprising that the model did not satisfy the Jorion's criteria on the capital market of Germany, although it is a developed financial market, unlike the others, which are emerging markets. The characteristics of the emerging market are not compatible with theoretical basis of the historical simulation. Hence, it is expected that in the case of these markets, the model performs poorly. BootstrapHS500 model, at the confidence level of 99%, satisfied the Jorion's criteria, also only in the case of the SAX index. At the confidence level of 99.5%, the model satisfied the Jorion's criteria also in case of three indexes: CROBEX10, SAX and BUX. However, a valid conclusion cannot be made based only on this criteria and the data shown in Table 1, about whether the bootstrap method brings an improvement of the applicability of the standard model of historical simulation, since the BootstrapHS500 model on some markets generated a lower and on other markets a higher number of VaR breaks, compared to the HS500 model.

In order to formally examine whether the tested models may be considered valid according to the Basel II standards, i.e to get the answer to the above question, it is necessary to use a formal test to examine the validity of the VaR models. For this purpose, in the paper we used the Kupiec's test of unconditional coverage and the Christofferson's test of conditional coverage. Both tests were used at 5% significance level, because a significance level of this magnitude generates clear evidence about the validity of the approach and implies that a model should be rejected only if the evidence against it is reasonably strong (Samanta, G.P. et al. (2010)). The Kupiec's unconditional coverage test results for the tested VaR models are given in Table 2.

Table 2. The results of Kupiec's unconditional coverage test

	HS500 VaR 99%		HS500 VaR 99, 5%		BootstrapHS500 VaR 99%		BootstrapHS500 VaR 99, 5%	
Stock index	Critical value	p-value	Critical value	p-value	Critical value	p-value	Critical value	p-value
BELEX15	2.082	0.149	3.949	0.047	0.844	0.358	1.837	0.175
CROBEX10	0.121	0.728	0.044	0.833	0.089	0.765	-	-
ATHEX20	2.082	0.149	0.413	0.520	3.700	0.054	3.949	0.047
IBEX35	2.082	0.149	3.949	0.047	3.700	0.054	6.574	0.010
DAX30	10.554	0.001	12.989	0.000	5.683	0.017	9.610	0.002
SAX	1.105	0.293	0.044	0.833	1.105	0.293	-	-
PX	0.844	0.358	3.949	0.047	0.844	0.358	1.837	0.175
BET	0.844	0.358	1.837	0.175	0.844	0.358	1.837	0.175
BUX	0.121	0.728	0.044	0.833	0.121	0.728	0.044	0.833

Source: Authors' calculations

As can be seen from Table 2, at the confidence level of 99%, the HS500 model did not satisfy

Christofferson-ovog testa uslovnog pokrića, moguće je da model koji nije zadovoljio test bezuslovnog pokrića prođe test uslovnog pokrića, ali to ne znači da je procena VaR validna. Rezultati Christofferson-ov testa uslovnog pokrića prikazani su u tabeli 3.

Tabela 3. Rezultati Christofferson-ovog testa uslovnog pokrića

	HS500 VaR 99%		HS500 VaR 99, 5%		BootstrapHS500 VaR 99%		BootstrapHS500 VaR 99, 5%	
Indeksi	kritična vrednost	p-vrednost	kritična vrednost	p-vrednost	kritična vrednost	p-vrednost	kritična vrednost	p-vrednost
BELEX15	12,990	0,002	8,024	0,018	0,116	0,943	1,837	0,399
CROBEX10	0,121	0,941	0,046	0,977	-	-	-	-
ATHEX20	2,082	0,353	0,413	0,813	0,832	0,660	3,949	0,139
IBEX35	12,990	0,002	8,024	0,018	2,060	0,357	6,574	0,037
DAX30	12,358	0,002	19,664	0,000	3,700	0,157	9,610	0,008
SAX	1,106	0,575	0,046	0,977	-	-	-	-
PX	0,844	0,656	3,949	0,139*	0,116	0,943	1,837	0,399
BET	0,844	0,656	1,837	0,399	0,116	0,943	1,837	0,399
BUX	0,121	0,941	0,046	0,977	1,118	0,572	0,046	0,977

Napomena: U slučajevima gde nije zabeležen klaster prekoračenja za izračunavanje racia verodostojnosti Markovljevog prvog reda korišćena je alternativna formula (detljnije videti Brandolini and Colucci, 2013).

*Uprkos prikazanoj p-vrednosti od 0,139, u slučaju PX indeksa, model HS500 nije prošao test uslovnog pokrića, s obzirom da nije zadovoljio kriterijum bezuslovnog pokrića na ovom tržištu.

Izvor: Autori

Kao što se može videti iz tabele 3, BootstrapHS500 model postigao je bolje perfromanse u odnosu na HS500 model. Za nivo poverenja od 99%, HS500 modela nije prošao test uslovnog pokrića na tržištu kapitala Srbije, Španije i Nemačke, dok za nivo poverenja od 99,5% nije prošao test u slučaju četiri indeksa i to: BELEX15, IBEX35, DAX30 i PX. Za nivo poverenja od 99% BootstrapHS500 model je prošao ovaj test na svim tržištima, dok za nivo poverenja od 99,5% nije zadovoljio ovaj kriterijum na tržištu kapitala Grčke, Španije i Nemačke. Imajući u vidu ove rezultate može se izneti zaključak da inkorporiranje bootstrap metode u standardni model istorijske simulacije doprinosi unapređenju aplikativnsti istorijske simulacije, u kontekstu zadovoljenja pravila validnosti, koja su definisana Bazel II standardom.

Zaključak

Imajući u vidu rezultate istraživanja, može se izvući zaključak da se aplikativnost istorijske simulacije može unaprediti u kontekstu zadovoljenja pravila validnosti definisana pravilima Bazel II standarda inkorporiranjem bootstrap metode u standardni model istorijske simulacije. Ovakav zaključak iznet je na osnovu rezultata Kupiec-ovog testa bezuslovnog pokrića.

Međutim, zbog ograničenja samog Kupiecovoc testa, kao i dobro poznate činjenice da kada nisu zadovoljene elementarne pretpostavke na kojima su model VaR izgrađeni, procene VaR neće biti validne, i u najboljem slučaju mogu da obezbede samo bezuslovno pokriće, u radu je korišćen i Christofferson-ov testa uslovnog pokrića. Rezultati ovog testa takođe svedoče u korist BootstrapHS500 model u odnosu na standardni model istorijske simulacije.

Naravno, prilikom prihvatanja i ovog zaključka treba biti oprezan jer Christofferson-ov test uslovnog pokrića ima sumnjivu statističku snagu za veličinu uzorka definisanu Bazel II standardom. Naime, asimptotski test sledi $\chi 2$ distribuciju sa dva stepena slobode pod nulte hipoteze da je serija pogodaka i prekoračenja sledi IID Bernoullivu raspodelu. U slučaju da se radi sa dovoljno velikim uzorcima, test će biti distribuiran kao $\chi 2$ sa dva stepena slobode. Međutim, u slučaju veličine uzorka definisanog Bazel II standardom snaga testa je diskutabilna.

the Kupiec's test only in the case of the capital market of Germany, while at the confidence level of 99.5%, the model did not pass this test on the four markets including: Serbia, Spain, Germany and Slovakia. On the other hand, the BootstrapHS500 model satisfies the criteria of the unconditional coverage test in all selected markets, at the confidence level of 99%, while at the confidence level of 99.5%, the model did not satisfy these criteria on the capital markets of Greece, Spain and Germany. Based on the results of the the Kupiec's unconditional coverage test, it can be concluded that incorporating the bootstrap method in the standard model of historical simulation contributes to the improvement of applicability of the historical simulation in the context of backtesting rules of the Basel II standards.

Theoretically speaking, the extreme losses, that occurred prior to and during the backtesting period, can cause VaR estimates to be very high and this way unconditional risk coverage is automatically achieved without taking into consideration the actual market risk. For this reason, for those markets on which the models passed the Kupiec's test, it is need to employ the Christoffersen's test. The results of the Christoffersen's conditional coverage test are given in Table 3.

As can be seen from Table 3, the BootstrapHS500 model had better performances than the HS500 model. At the confidence level of 99%, the HS500 model did not satisfy the test of conditional coverage in the case of the capital markets of Serbia, Spain and Germany, while at the confidence level of 99.5%, the model did not pass this test in case of four indexes: BELEX15, IBEX35, DAX30 and PX. At the confidence level of 99%, the BootstrapHS500 model passed the test in all markets, while at the confidence level of 99.5%, the model did not satisfy this at the capital markets of Greece, Spain and Germany. Based on the results of the research in this paper it can be concluded that incorporating the bootstrap method in the standard model of historical simulation brings the improvement of applicability of the historical simulation in the context of the backtesting rules of the Basel II standards.

Conclusion

According to the presented results it can be concluded that incorporating the bootstrap method in the standard model of historical simulation brings the improvement of the historical simulation's applicability in the context of backtesting rules of the Basel II

Table 3.The results of the Christoffersen's conditional coverage test

	HS500 VaR 99%		HS500 VaR 99, 5%		BootstrapHS500 VaR 99%		BootstrapHS500 VaR 99, 5%	
Indices	Critical value	p-value	Critical value	p-value	Critical value	p-value	Critical value	p-value
BELEX15	12.990	0.002	8.024	0.018	0.116	0.943	1.837	0.399
CROBEX10	0.121	0.941	0.046	0.977	-	-	-	-
ATHEX20	2.082	0.353	0.413	0.813	0.832	0.660	3.949	0.139
IBEX35	12.990	0.002	8.024	0.018	2.060	0.357	6.574	0.037
DAX30	12.358	0.002	19.664	0.000	3.700	0.157	9.610	0.008
SAX	1.106	0.575	0.046	0.977	-	-	-	-
PX	0.844	0.656	3.949	0.139*	0.116	0.943	1.837	0.399
BET	0.844	0.656	1.837	0.399	0.116	0.943	1.837	0.399
BUX	0.121	0.941	0.046	0.977	1.118	0.572	0.046	0.977

Note: In the cases where the sample has $T_{11} = 0$ (there are no consecutive VaR breaks), an alternative formula was used in the paper to calculate the first-order Markov likelihood (see Brandolini and Colucci, 2013).

*Despite p-value of 0.139, in the case of PX index, model HS500 did not pass the test of conditional coverage, since it did not meet the criteria of unconditional coverage on this market.

Source: Authors' calculations

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standards. This conclusion is confirmed by the results of the Kupiec's test.

However, due to the drawbacks of the Kupiec's test as well as the well-known fact that when elementary assumptions of most VaR approaches are not satisfied, the VaR estimates will be unreliable and, at best, they can only provide unconditional coverage, Christoffersen's test of conditional coverage was also used in the paper. The results of this test, also, testify in favor of the BootstrapHS500 model over the standard model of historical simulation.

Nevertheless, this conclusion should also be accepted with caution given that Christoffersen's test of conditional coverage has a questionable statistical power for the sample size defined by the Basle Accord. The test is asymptotically distributed as $\chi 2$ with two degrees of freedom under the null hypothesis that the hit sequence is IID Bernoulli with the mean equal to the VaR coverage rate. In the case of a large enough sample, the test will be distributed as a $\chi 2$ with two degrees of freedom. However, if the sample size is defined by Basel II standards, the power of the test is disputable.